

DEPARTMENT OF MATHEMATICS

Course Code				23MAMAL231			
Title of the Course				GROUP THEORY			
Offered to: (Programme/s)				B.Sc Hons (Mathematics)			
L	5	T	0	P	0	C	4
Year of Introduction:		2024-25		Semester:			3
Course Category:		MAJOR		Course Relates to:		GLOBAL	
Year of Revision:		2024		Percentage:		NA	
Type of the Course:				SKILL ENHANCED			
Crosscutting Issues of the Course :				GENDER			
Pre-requisites, if any				Basics of Algebraic Structures			

Legend:

Offered to: Name of the Program (B.Sc. Hons Mathematics)

Category: Major

Course Relates to: Global

Type of the Course: Skill development

Crosscutting Issues of the Course: Group, Properties of Groups and algebraic structures.

L: Lecture; T: Tutorial; P: Practicum/Practical/Project; C: Credits

Course Description:

Abstract algebra is a branch of mathematics that studies algebraic structures such as groups, subgroups, Normal Subgroups, Homomorphisms, Permutations Groups & Cyclic groups and more abstract constructs. Unlike elementary algebra, which primarily deals with manipulating symbols and solving equations, abstract algebra focuses on understanding the fundamental properties and relationships between these algebraic structures.

Course Aims and Objectives:

S.NO	COURSE OBJECTIVES
1	Classify different types of groups and study their properties. Groups are fundamental algebraic structures that arise in various mathematical contexts, and understanding their classifications helps in identifying common structures and relationships among different mathematical objects.
2	Identifying and studying subgroups, mathematicians can understand the internal organization of a group, including its symmetries and algebraic properties.
3	Understand the relationships between different groups and their representations, facilitating a deeper understanding of symmetry, algebraic properties, and abstract structures.
4	Show relationships between objects, analyzing kernels and images, facilitating factorization and decomposition of algebraic structures, supporting representation theory and connecting algebraic concepts with geometry.
5	Apply knowledge to identify and characterize various algebraic properties within the permutation groups.

Course Outcomes

At the end of the course, the student will be able to...

CO NO	COURSE OUTCOME	BTL	PO	PSO
CO1	Understand concepts of groups and its properties	K3	1	1
CO2	Define subgroups and determine the given subsets of a group are sub groups.	K1	1	1

CO3	Explain the significance of cosets, normal subgroups and factor groups	K2	1	1
CO4	Explain group homomorphisms and isomorphisms	K2	1	1
CO5	Find cycles of a given permutation and understand the properties of cyclic groups.	K4	2	2

For BTL: K1: Remember; K2: Understand; K3: Apply; K4: Analyze; K5: Evaluate; K6: Create

CO-PO MATRIX									
CO NO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PSO1	PSO2
CO1	1							1	
CO2	1							1	
CO3	1							1	
CO4	1							1	
CO5	2								2

Course Structure:

UNIT-I : GROUPS

Binary Operation, Semi group, Algebraic Structure, Monoid, Cancellation laws, Group definition, Abelian group, Elementary Properties, Finite and Infinite groups with examples, Order of a group with examples, Addition modulo m – Definition – theorem – Problems, Multiplication Modulo P – definition- $\{1, 2, 3, \dots, p-1\}$ where P is a prime number is a group – theorem – Problems, Order of an element of a group – Definition – Theorems.

Description: This unit familiarizes the students, the concept of Group. Groups are fundamental structures that capture the essence of symmetry and transformation. A group consists of a set of elements along with a single binary operation that combines any two elements to produce another element in the group.

Examples/Applications/Case Studies:

1. Explain axioms of a group, and find whether the given set is satisfying the properties or not.
2. Distinguish the given set is a Group or Abelian Group.
3. Extend all possible order of elements in the given set.
- 4.

Exercises:

1. Show that if every element of the group G is its own inverse, then G is abelian.
2. Prove that the given set is abelian group.
3. Find the order of any element of the group.

Learning Outcomes:

1. Develop an intuition for identifying and analyzing symmetries in different contexts, from geometric shapes to abstract algebraic structures.
2. Understanding the properties of groups (closure, identity element, inverses, associativity) reinforces foundational concepts in algebra and helps build skills in manipulating abstract structures.
3. Practicing with different group operations enhances problem-solving abilities and logical reasoning.

Web Resources:

1. **Online Math Notes - Groups:** <chrome-extension://efaidnbmnnnibpcajpcglclefindmkaj/https://www.jmilne.org/math/CourseNotes/GT.pdf>
2. **Youtube Videos by AVM Math Hub:** https://www.youtube.com/watch?v=BpnMr8bVw-8&list=PLp1ohDz4ATf6NohF0n8bLx843Hxr_f_Bw&index=1

UNIT-II: SUB GROUPS

Complex definition, Multiplication of two complexes, Inverse of a complex, subgroup definition, Identity and Inverse of a subgroup, Criterion for a complex to be a subgroup, Criterion for the product of two subgroups to be a subgroup, Union and Intersection of subgroups, Cosets Definition – Properties of cosets, Index of a subgroups of a finite groups, Lagrange’s Theorem.

Description: A subgroup is a set that is a subset of a larger group and that itself forms a group with the same operation as the larger group. Understanding subgroups is crucial in various areas of mathematics because they often reveal underlying structures and symmetries within a larger group.

Examples/Applications/Case Studies:

1. Find how many subgroups of order 5.
2. Find the number of Proper subgroups of the given set (Z_6).
3. How to find two left or right cosets of a subgroup is disjoint or identical.

Exercises:

1. Show that Intersection of two subgroups is again a subgroup.
2. How to show a bijection between any two left cosets in a subgroup of a group.

Learning Outcomes:

1. Understand the concept of subgroup.
2. Determine whether a subset of a group is a subgroup of that group by applying the Subgroup Test.
3. Determine whether a set with a given binary operation is or is not a group.

Web Resources:

1. Subgroups problems and solutions
<https://byjus.com/maths/subgroups/>
2. YouTube videos by AVM Math hub (Student connect)
https://www.youtube.com/watch?v=RuNsEKt6oVw&list=PLp1ohDz4ATf6NohF0n8bLx843Hxr_f_Bw&index=41

UNIT-III: NORMAL SUBGROUPS

Definition of a normal subgroup, Proper and improper normal subgroups, Intersection of two normal subgroups, Subgroup of index 2 is a normal subgroup, Simple group, Quotient group, Criteria for the existence of a Quotient group.

Description: Normal subgroups are a special type of subgroup with key properties that play a central role in group theory. Normal subgroups are fundamental to many aspects of group theory, providing a way to study and understand the structure of groups through quotient groups and contributing to the broader theory of homomorphisms and group actions.

Examples/Applications/Case Studies:

1. Find the difference between trivial subgroup and whole group.
2. How to find the centre of the group.
3. Show that In an abelian group, every subgroup is normal.

Exercises:

1. Prove that every finite group of order 72 is not a simple group.
2. Prove that Every Quotient Group of an Abelian Group is Abelian.

Learning Outcomes:

1. Determine whether a subgroup of a group is normal.
2. Determine the set G/H of distinct left cosets of H in G .
3. Determine the quotient group G/N of G by a normal subgroup N .

Web Resources:

1. **Online Math Notes – Normal Subgroups :** <chrome-extension://efaidnbmnnnibpcajpcglclefindmkaj/https://egyankosh.ac.in/bitstream/123456789/71714/1/Unit-6.pdf>
2. **YouTube videos by AVM Math hub (Student connect):**
https://www.youtube.com/watch?v=aQ0w6B2N7_g&list=PLp1ohDz4ATf6NohF0n8bLx843Hxr_f_Bw&index=29

UNIT-IV: HOMOMORPHISM

Definition of a Homomorphism, Image of a Homomorphism, Properties of a Homomorphism, Isomorphism, Automorphism definitions and elementary properties, Kernel of a homomorphism, Fundamental theorem on homomorphism of groups and Applications, Inner automorphism, outer automorphism.

Description: a homomorphism is a fundamental concept that describes a structure-preserving map between two algebraic structures. Here's a detailed description of homomorphisms, focusing on their role and properties in group theory.

Examples/Applications/Case Studies:

1. Identifying when two groups are structurally the same (isomorphic). This helps in classifying and studying groups up to isomorphism.
2. Show that the given function is homomorphism then G is abelian.

Exercises:

1. Show that the function f is homomorphism if and only if G is commutative.
2. Prove that the set of all automorphisms of a group G forms a group with respect to composition of mapping.

Learning Outcomes:

1. Understand the concept of homomorphism.
2. Learn to use homomorphisms to solve problems involving algebraic structures.

Web Resources:

1. Online Math Notes – Group Theory:

[extension://efaidnbmnnnibpcajpcglclefindmkaj/https://people.bath.ac.uk/gt223/MA30237/lnotes.pdf](https://people.bath.ac.uk/gt223/MA30237/lnotes.pdf)

2. YouTube videos by AVM Math hub (Student connect):

https://www.youtube.com/watch?v=fdjWzISYkGw&list=PLp1ohDz4ATf6NohF0n8bLx843Hxr_f_Bw&index=36

UNIT-V: PERMUTATIONS AND CYCLIC GROUPS

Definition of a permutation group, Equal permutations, Permutation multiplications, Order of a permutation, Inverse of a permutation, Orbits and cycles of permutation, Transposition, even and odd permutations – Theorem – Related Problems, Cayley's theorem – Related Problems, Definition of a cyclic group – Properties of Cyclic group, Standard theorems on cyclic groups – related problems.

Description: Permutation groups are fundamental mathematical structures within the realm of group theory, a branch of abstract algebra. Cyclic groups are a fundamental concept in abstract algebra, specifically within the broader field of group theory. They are named "cyclic" because they are generated by a single element, akin to how rotations around a circle produce cyclic patterns.

Examples/Applications/Case Studies:

1. Show that the given permutations are even or odd.
2. Evaluate the product of two permutations.
3. Show that the given set is a cyclic group with one of the element as a generator.

Exercises:

1. Show that every cycle of length greater than 2 can be expressed as a product of transpositions.
2. Show that the given set is a cyclic group with the given multiplication modulo and also find the number of generators.

Learning Outcomes:

1. Define and understand the properties of Permutation groups, symmetric groups, alternating groups.
2. Solve problems involving the permutation groups, symmetric groups and alternating groups.
3. Extend group structure to finite permutation groups.

Web Resources:

1. Online web notes: Permutation Groups.

<https://math.umd.edu/~immortal/MATH403/lecturenotes/ch5.pdf>

2. YouTube videos by AVM Math hub (Student connect):

<https://www.youtube.com/watch?v=XuYsWgwmqt4>

TEXT BOOKS :

1. Venkateswara Rao V, Sharma B.V.S.S & Anjaneya Sastry S. (2015). *A textbook of Mathematics (Abstract Algebra) - Vol.- I* (2nd Edition). S – Chand.

REFERENCE BOOKS:

1. Dr Anjaneyulu A. (2015). *A textbook of Mathematics (Abstract Algebra) - Vol- I* (2nd Edition). Deepthi Publications.
2. Khanna M.L. (2012). *Modern Algebra* (20th Edition). Jai Prakash Nath & Co.

Model Paper

Course Code: 23MAMAL231

Time: 3Hrs

Offered to: B.Sc HONS (MATHEMATICS)

Title of the Course: GROUP THEORY

MAX MARKS: 70

SECTION – A

I Answer the following

5 x 4 = 20M

1. (a) Show that Identity element is unique. (CO1,K1)
 (OR)
 (b) Show that in a group G for $a, b \in G$, $(ab)^2 = a^2b^2 \Leftrightarrow G$ is abelian. (CO1,K1)
2. (a) If H_1, H_2 are two subgroups of a group G, then show that $H_1 \cap H_2$ is also a subgroup of G. (CO2,K2)
 (OR)
 (b) If H is a subgroup of a group G and $a \in G$, then show that $a \in H \Rightarrow aH = H$ (CO3,K2)
3. (a) If N, M are normal subgroups of G, then NM is also a normal subgroup of G. (CO3,K1)
 (OR)
 (b) Prove that every group of prime order is simple. (CO3,K2)
4. (a) Prove that every homomorphic image of an abelian group is abelian. (CO4,K1)
 (OR)
 (b) If for a group G, $f : G \rightarrow G$ is given by $f(x) = x^2, \forall x \in G$ is a homomorphism, prove that G is abelian. (CO4,K2)
5. (a) Examine whether $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 3 & 1 & 8 & 5 & 6 & 2 & 4 \end{pmatrix}$ is even or odd. (CO5,K4)
 (OR)
 (b) Find the number of generators of a cyclic group of order 15. (CO5,K1)

SECTION – B

II Answer the following

5 x 10 = 50M

- 6(a) Prove that the set of nth roots of unity under multiplication form a finite group. (CO1,K3)
 (OR)
 (b) Show that set Q_+ of all positive rational numbers forms an abelian group under the composition defined by “o” such that $aob = \frac{ab}{3}, for a, b \in Q$ (CO1,K3)
- 7 (a) Prove that H is a non – empty complex of a group G. The necessary and sufficient condition for H to be subgroup of G is $a.b \in H \Rightarrow ab^{-1} \in H$ where b^{-1} the inverse of b in G. (CO2,K3)
 (OR)
 b) State and Prove Lagrange’s Theorem. (CO3,K4)
- 8 (a) Prove that A subgroup H of a group G is a normal subgroup of G iff each left coset of H in G is a right coset of H in G. (CO3,K3)
 (OR)
 (b) State and Prove Quotient Group Theorem. (CO3,K4)
- 9 (a) The necessary and sufficient condition for a homomorphism f of a group G onto a group G' with kernel K to be an isomorphism of G into G' is that $K = \{e\}$. (CO4,K3)
 (OR)
 (b) State and Prove Fundamental Theorem on Homomorphism of Groups. (CO4,K4)
- 10 (a) If a cyclic group G is generated by an element a of order n, then a^m is a generator of G iff $(m, n) = 1$. (CO5,K4)
 (OR)
 (b) If $f = (1 \ 2 \ 3 \ 4 \ 5 \ 8 \ 7 \ 6), g = (4 \ 1 \ 5 \ 6 \ 7 \ 3 \ 2 \ 8)$ are cyclic permutations, then show that $(fg)^{-1} = g^{-1} f^{-1}$. (CO5,K3)

SRI DURGA MALLESWARA SIDDHARTHA MAHILA KALASALA, VIJAYAWADA - 10.

(An autonomous college in the jurisdiction of Krishna University, Machilipatnam, A.P. India.)

Course Code				23MAMAL232			
Title of the Course				NUMERICAL METHODS			
Offered to: (Programme/s)				B.Sc Hons (Mathematics)			
L	5	T	1	P	0	C	4
Year of Introduction:		2024-25		Semester:			3
Course Category:		MAJOR		Course Relates to:		GLOBAL	
Year of Revision:				Percentage:		NA	
Type of the Course:				SKILL ENHAMCED			
Crosscutting Issues of the Course :				GENDER			
Pre-requisites, if any				Advanced calculus and Differential equations			

Legend:**Offered to:** Name of the Program (B.Sc. Hons Mathematics)**Category:** Major**Course Relates to:** Global**Type of the Course:** Skill development**Crosscutting Issues of the Course:** Errors due to the approximations made during the discretization and the use of iterative computations.

L: Lecture; T: Tutorial; P: Practicum/Practical/Project; C: Credits

Course Description:

This course is a basic course offered to UG student of Engineering/ Science background. It contains Interpolation, Numerical differentiation, and curve fitting. It plays an important role for solving various Engineering Sciences problems. Therefore, it has tremendous applications in diverse fields in engineering sciences.

Course Aims and Objectives:

S.NO	COURSE OBJECTIVES
1	Define the Basic concepts of operators Δ, ∇, E and solving symbolic relations.
2	Apply numerical methods with equal intervals and unequal intervals to find out solution of algebraic equations using different methods under different conditions .
3	Apply various central difference interpolation methods to find out the solutions of the given functions.
4	Able to Calculate the value of the derivative of a function at some assigned value using different methods.
5	Improve the accuracy and reliability of a model or system by reducing the error between the data output.

Course Outcomes

At the end of the course, the student will be able to...

CO NO	COURSE OUTCOME	BTL	P O	PSO
CO1	Understand concepts of operators and relation between the operators.	K2	1	2
CO2	Calculate the value of a function using interpolation.	K5	2	2
CO3	Define the Basic concepts of operators \square, \square and solve problems using central difference formulas.	K2	2	2
CO4	Find derivatives using various numerical methods.	K3	2	2
CO5	Fit polynomials to given set of points.	K5	2	2

CO-PO MATRIX									
CO NO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PSO1	PSO2
CO1	2								2
CO2		2							2
CO3		2							2
CO4		2							2
CO5		2							2

Course Structure:

UNIT-I: THE CALCULUS OF FINITE DIFFERENCES

The operators Δ , ∇ , E , Fundamental theorem of difference calculus, properties of Δ , ∇ , E and problems on them to express any value of the function in terms of the leading terms and the leading differences, relations between E and D , relation between D and Δ , Problems on one or more missing terms, Factorial notation, Problems on separation of symbols, Problems on Factorial notation.

Examples/Applications/Case Studies:

1. Explain fundamental theorem of difference calculus.
2. Find the factorial form of a given function.
3. Find the missing terms of the given data using fundamental theorem.

Exercises:

1. Show that the operators Δ and E are commutative.
2. Evaluate $\Delta^n \sin(ax+b)$.
3. Construct the forward difference table for the given data.

Learning Outcomes:

4. Understand various finite difference concepts and interpolation methods.
5. Evaluation of missing terms using fundamental theorem of difference calculus.
6. Compute various relations between the difference operators.

Web Resources:

1. <http://spartan.ac.brocku.ca/~jvr/bik/MATH2P20/notes.pdf>
2. <http://www.math.iitb.ac.in/~baskar/book.pdf>

UNIT-II: INTERPOLATION WITH EQUAL AND UNEQUAL INTERVALS

Derivations of Newton Gregory Forward and Backward interpolation and problems on them, Divided differences, Newton divided difference formula and problems, Lagrange's interpolation formula and problems.

Description: Interpolation is a method in numerical analysis that estimates intermediate values of a function based on a set of known data points. It can be used in engineering and science to estimate the value of a function for values of the independent variables that are not explicitly given in the data.

Examples/Applications/Case Studies:

4. Find the approximate value of $f(x)$ for the given data using suitable interpolation formula..
5. Using the Newton's divided difference formula, find a polynomial function for the given data.
6. By Lagrange's interpolation formula find the value of y at some x for the given data.

Exercises:

3. Derive the Newton's forward interpolation formula for equal intervals.
4. State and prove the Newton's divided difference formula for unequal intervals.
5. Derive Lagrange's interpolation formula.

Learning Outcomes:

4. Understand the subject of various numerical methods that are used to obtain approximate solutions.
5. Determine a polynomial form of a function using various numerical methods.
6. Analyze the applicability of numerical method for the given data.

Web Resources:

3. <https://archive.nptel.ac.in/courses/111/107/111107062/>
4. https://www.lkouniv.ac.in/site/writereaddata/siteContent/202004032250571912siddharth_bhatt_engg_Interpolati

UNIT-III: CENTRAL DIFFERENCE INTERPOLATION FORMULA

Central difference operators δ, μ, σ and relation between them, Gauss's Forward interpolation formulae, Gauss's backward interpolation formulae, Stirling's formula, Bessel's formula and problems on the above formulae.

Description: The central difference interpolation formula is a statistical method in numerical analysis that uses unknown values to model complicated functions and estimate unknown values.

Examples/Applications/Case Studies:

4. Apply any suitable central difference formula to approximate value of the function y for the given data.
5. Use Gauss's interpolation formula to find y at some x with the help of given data.
6. Explain any relation between central difference operators.

Exercises:

3. Derive the Gauss's forward central difference formula.
4. State and prove Stirling's central difference formula.
5. Derive Bessel's formula.

Learning Outcomes:

4. Learn to code a numerical method in a modern computer language.
5. Obtain the best accuracy methods, and can be very effective, and close to the exact value for several problems.

Web Resources:

3. https://math.iitm.ac.in/public_html/sryedida/caimna/interpolation/cdf.html
4. <https://archive.nptel.ac.in/courses/111/107/111107105/>

UNIT-IV: NUMERICAL DIFFERENTIATION

Derivatives using Newton's forward difference formula, Newton's backward difference formula, Derivatives using central difference formula, Stirling's interpolation formula, Newton's divided difference formula.

Description: Numerical differentiation is the process of finding the numerical value of a derivative of a given function at a given point. In business differentiation is used to find profit and loss for the future of investment using graphs. Temperature variations are also calculated by using differentiation.

Examples/Applications/Case Studies:

3. Calculate the highest and lowest point of a curve in a graph.
4. Compute the first and second derivative of a function f from given values of f .

Exercises:

3. Derive the formulas for first derivative and second derivative using Newton's backward formula.
4. Find the approximate value of second derivative using given set of paired values.

Learning Outcomes:

3. Demonstrate understanding of common numerical methods and how they are used to obtain approximate solutions to otherwise intractable mathematical problems.
4. Understand the applicability and limitations of Numerical schemes.

Web Resources:

3. <https://archive.nptel.ac.in/content/storage2/courses/122104018/node117.html>
4. https://onlinecourses.nptel.ac.in/noc19_ma21/preview

UNIT-V: CURVE FITTING

Method of least squares, Fitting of a straight line, Non linear curve fitting, curve fitting by a sum of exponentials.

Description: Curve fitting is a process that involves finding a function or curve that best fits a set of data points. The curve fitting process can involve constraints on the data points. The goal is to find a function that minimizes the residual or distance between the data points and the function. A smaller residual means a better fit.

Examples/Applications/Case Studies:

4. To find the path of roller coaster, the turning point of a railway track, paths of a road particularly in hilly areas.
5. Used to estimate variable values between data samples and outside the data sample range.

Exercises:

3. Fit a straight line for the given paired values by the method of least squares technique.
4. Fit a second degree parabola for the given data.

Learning Outcomes:

4. Estimate the variable value between data samples or outside the data sample range.
5. Find mathematical relationships to perform further data processing, such as error compensation, velocity and acceleration calculation.
6. Learn how to reduce noise and smmothout origina data points.

Web Resources:

1. <https://perhuaman.files.wordpress.com/2014/07/metodos-numericos.pdf>
2. <https://archive.nptel.ac.in/content/storage2/courses/122104019/numerical-analysis/Rathish-kumar/least-square/r1.htm>

Text Books :

1. Gupta and Malik, Calculus of Finite Differences and Numerical Analysis. Krishna Prakashan Mandir, Meerut.

Reference Books:

- 1 S.S. Sastry, Introductory Methods of Numerical Analysis, Prentice Hall of India Pvt. Ltd., NewDelhi-110001, 2006.
- 2 P. Kandasamy, K. Thilagavathy, Calculus of Finite Differences and Numerical Analysis. S. Chand & Company, Pvt. Ltd., Ram Nagar, New Delhi-110055.

Course Code: 23MAMAL232

Time: 3Hrs

Offered to: B.Sc.Hons. Mathematics

Title of the Course: NUMERICAL METHODS

MAX MARKS: 70

Model Paper

SECTION – A

I Answer the following

5 x 4 = 20M

1. (a) Construct the forward difference table for the following data **(CO1,K1)**

x	1	2	3	4	5
F(x)	7	12	29	64	123

(OR)

(b) Express $f(x) = x^4 - 4x^3 + 7x^2 + 3x - 6$ in terms of the factorial notation. **(CO1,K1)**

2. (a) Use Lagranges interpolation formula to find the form of the function from the data **(CO2,K3)**

x	0	3	4
F(x)	12	6	8

(OR)

(b) Construct the Newton' divided difference table for the given data: **(CO2,K3)**

x	1	2	7	8
F(x)	1	5	5	4

3. (a) Prove that $\sqrt{1 + \delta^2 u^2} = 1 + \frac{1}{2} \delta^2$ **(CO3,K2)**

(OR)

(b) Derive Bessel's formula. **(CO3,K2)**

4. (a) Write Newton's forward difference formula to find $\frac{dy}{dx}$ at $x = x_0$. **(CO4,K1)**

(OR)

(b) Using the given table find $\frac{dy}{dx}$ at $x = 2.03$ **(CO4,K1)**

x	1.96	1.98	2.00	2.02	2.04
y	0.7825	0.7739	0.7651	0.7563	0.7473

5(a) Fit a polynomial of the second degree to the data points

(CO5,K2)

x	0	1	2
y	1	6	17

(OR)

(b) By the method of least squares fit a straight line to the following data

(CO5,K2)

x	1	2	3	4	5
y	14	27	40	55	68

SECTION – B

II Answer the following

5 x 10 = 50M

6(a) State and Prove fundamental theorem of difference calculus.

(CO1,K3)

(OR)

(b) Obtain the missing terms from the following data:

(CO1,K3)

x	1	2	3	4	5	6	7	8
f(x)	1	8	?	64	?	216	343	512

7(a) State and Prove Newton Gregory Forward interpolation formula.

(CO2,K3)

(OR)

b) State and Prove Lagrange's Interpolation formula.

(CO2,K3)

8(a) State and prove Gauss's forward interpolation formula.

(CO3,K3)

(OR)

(b) Apply Bessel's formula to find the value of f (27.4) from the table

(CO3,K4)

x:	25	26	27	28	29	30
f(x):	4.000	3.846	3.704	3.571	3.448	3.333

9(a) Using the given table find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.2$

(CO4,K4)

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

(OR)

(b) Derive the formulas of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ using Newtons mackward interpolation formula. (CO4,K4)

10. (a) Fit a power curve $y = ae^{bx}$ by the method of least squares

(CO5,K4)

x	0.5	1	1.5	2	2.5
y	0.45	2.15	9.15	40.35	180.75

(OR)

(b) Fit a curve $y = ax^b$ by the method of least squares using the following table

(CO5,K4)

x	61	26	7	2.6
y	350	400	500	600

SRI DURGA MALLESWARA SIDDHARTHA MAHILA KALASALA, VIJAYAWADA - 10.
(An autonomous college in the jurisdiction of Krishna University, Machilipatnam, A.P. India.)

Course Code				23MAMAL233			
Title of the Course				Laplace transforms			
Offered to: (Programme/s)				Bsc.Hons(Mathematics)			
L	5	T	0	P	0	C	4
Year of Introduction:		2024-25		Semester:		3	
Course Category:		Major		Course Relates to:		Local	
Year of Revision:				Percentage:		NA	
Type of the Course:				Skill Enchanced			
Crosscutting Issues of the Course :							
Pre-requisites, if any							

Legend:

Offered to: Name of the Program (B.Com Hons/ B.A Hons/B.Sc. Hons/B.C.A Hons/ B.B.A Hons)

Category: Major/Minor/MDC/SDC/ENG/TEL/VAC

Course Relates to:Local, Regional, National, Global

Type of the Course:Employability/ Entrepreneurship/ Skill development

Crosscutting Issues of the Course: Gender, Environment and Sustainability, Human Values and Professional Ethics

L: Lecture; T: Tutorial; P: Practicum/Practical/Project; C: Credits

Course Description:

An overview of the course content and objectives.

Laplace transform is a fundamental tool in integral calculus.it is used to solve various types of differential equations, difference equations, integral equations etc. which arise naturally in engineering and basic sciences

Course Aims and Objectives:

S.NO	COURSE OBJECTIVES
1	Knowledge in fundamental concepts of Laplace transforms of a function
2	Ability to develop necessary skills to recognize the properties of Laplace transforms their applications
3	Understand Laplace transforms of various functions
4	Ability to understand the basic concepts of inverse transforms of a functions
5	Competence in concepts of convolution theorem, Heaviside expansion and their applications

Course Outcomes

At the end of the course, the student will be able to...

CO NO	COURSE OUTCOME	BT L	P O	PS O
CO1	Understand the definition and properties of Laplace transformations	K1	1	2
CO2	Get an idea about first and second shifting theorems and change of scale property	K2	1	2
CO3	Analyse the Laplace transform of various functions	K4	2	2
CO4	Know the reverse transformation of Laplace and its properties	K2	2	2

CO5	Get the knowledge of application of convolution theorem	K3	2	2
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For BTL: K1: Remember; K2: Understand; K3: Apply; K4: Analyze; K5: Evaluate; K6: Create

CO-PO MATRIX									
CO NO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PSO1	PSO2
CO1	3								2
CO2	2								2
CO3		2							2
CO4		2							2
CO5		2							2

Use the codes 3, 2, 1 for High, Moderate and Low correlation Between CO-PO-PSO respectively

Course Structure:

Unit-1

Laplace transforms-1

15 hrs

Definition of Laplace transform- linearity property-piecewise continuous functions-Existence of Laplace transforms- functions of exponential order and of class A, First shifting theorem.

Description: This unit familiarizes the students, the concept of Laplace transforms. The Laplace transform is used to solve differential equations.

Examples/Applications/Case Studies:

5. Explain how to apply Laplace transforms for given functions.
6. Extend the different properties of Laplace transforms can be applied or not
7. Combining some of these simple Laplace transforms with the properties of the Laplace transforms

Exercises:

4. Find $L(\sin 2t \cos 3t)$
5. Find the Laplace transform of the function $F(t) = 4, 0 < t < 1$
 $3, t > 1.$
6. Evaluate $L(\sin t \cos t)$

Learning Outcomes:

7. Learn the application of Laplace transform in engineering analysis.
8. Understanding the properties of Laplace transforms reinforces foundational concepts in integral transforms
9. Learn the required conditions for transforming variable or variables in functions by the Laplace transform

Web Resources:

3. **Online Math Notes – Laplace transforms -I:**
https://sist.sathyabama.ac.in/sist_coursematerial/uploads/SMT1401.pdf

4. **Youtube Videos :**

https://www.youtube.com/watch?v=5hPD7CF0_54

Unit-2: Laplace Transforms-II

Second Shifting Theorem, Change of Scale Property, Laplace transform of the derivative of $f(t)$, Initial value theorem and Final value theorem.

Description: The second shift theorem is similar to the first except that, in this case, it is the time variable that is shifted not the t - variable

Examples/Applications/Case Studies:

7. If $L(F(t)) = \frac{9p^2 - 12p + 15}{(p-1)^3}$ then find $L(F(3t))$ using change of scale property
8. Evaluate $L(G(t))$ where $G(t) = \begin{cases} \cos(t - \frac{\pi}{3}), t > \frac{\pi}{3} \\ 0, t < \frac{\pi}{3} \end{cases}$
9. State and prove initial value theorem

Exercises:

6. Find the Laplace transform of $\cos at$ using the theorem on transforms of derivatives
7. Find $L(\sin^2(at))$ by using change of scale property

Learning Outcomes:

7. Understand the properties of Laplace transform
8. Determine how to apply the properties to given functions.
9. Will able to solve the change of scale, second shifting, transform of derivatives problems

Web Resources:

1. Laplace transform -II
<https://byjus.com/maths/Laplace-transform/>
2. YouTube videos :
<https://www.youtube.com/watch?v=C0GPZZ15Shk>

Unit-3:Laplace Transforms-III

Laplace transform of Integrals and related problems– Multiplication by t , Multiplication by t^n and related problems- division by t and related problems- Evaluation of integrals by Laplace transforms

Description: The Laplace transform's key property is that it converts differentiation and integration in the time domain into multiplication and division by t in the Laplace domain. The Laplace transform is an integral transform that can be used to evaluate integrals, it converts a function of real variable into a function of a complex variable.

Examples/Applications/Case Studies:

1. Find $L\left(\frac{e^{-at} - e^{-bt}}{t}\right)$
2. Evaluate $L(t \cos 3t)$
3. Show that $\int_0^{\infty} t e^{-3t} \sin t \, dt = \frac{3}{50}$

Exercises:

1. Find the Laplace transform of $\frac{1 - \cos t}{t^2}$
2. Find $L((t^2 - 3t + 2)\sin 3t)$
3. Evaluate $\int_0^{\infty} t e^{-3t} \, dt$

Learning Outcomes:

6. Will able to know that how to apply the multiplication by t and division by t
7. Will able to understand the concepts of division by t and evaluation of integrals of Laplace transforms.
8. Able to solve the problems of Laplace transforms

Web Resources:

5. Online Math Notes- Laplace transform-III: https://mathalino.com/reviewer/advance-engineering-mathematics/evaluation-integrals#google_vignette
6. YouTube videos : <https://www.youtube.com/watch?v=sIGB8nKddUw>

Unit -4: Inverse Laplace transforms –I

Definition of inverse Laplace transform – linearity property - First shifting Theorem-Second Shifting Theorem- Change of Scale Property- use of partial fractions – examples

Description: The inverse Laplace transform is used to find the original function from its Laplace transform. It's a powerful tool for solving non homogeneous linear differential equations which are equations where the solution to the derivative is not zero

Examples/Applications/Case Studies:

1. Find the inverse Laplace transform of $\frac{2p-5}{p^2-9}$
2. Evaluate inverse Laplace transform of $\left(\frac{p+1}{p^2+6p+25}\right)$

Exercises:

1. Find the inverse Laplace transform of $\frac{e^{-3p}}{(p-2)^2}$
2. Find the inverse Laplace transforms of $\frac{3p+1}{p^2-2p-3}$ by partial fractions

Learning Outcomes:

5. Understand the concept of inverse Laplace transforms.
6. Learn to use inverse Laplace transform to solve problems.

Web Resources:

5. **Online Math Notes – inverse Laplace transform –I:** <https://www.vedantu.com/maths/inverse-Laplace-transform>
6. **YouTube videos :** <https://www.youtube.com/watch?v=Y8GXpS31CGI>

Unit -5: Inverse Laplace transforms –II

Inverse Laplace transforms of derivatives- inverse Laplace transforms of Integrals- multiplication by powers of 'p'- Division by powers of 'p'-convolution Definition- Convolution theorem –Proof and Applications- Heaviside's expansion theorem and its Applications

Description: The convolution theorem for Laplace transforms states that taking the convolution of two functions and then taking the Laplace transform is the same as taking the Laplace transform of each function separately and then multiplying the two Laplace transform together

Examples/Applications/Case Studies:

1. Find $L^{-1}\left(\frac{1}{p}\log\left(\frac{p+2}{P+1}\right)\right)$ by division by 'p'
2. Find $L^{-1}\left(\frac{P+3}{(p^2+6p+13)^2}\right)$

Exercises:

3. Using Heaviside's expansion formula, Find $L^{-1}\left(\frac{3p+1}{(p-1)(p^2+1)}\right)$
4. Using convolution theorem, $L^{-1}\left(\frac{1}{p(p+1)(p+2)}\right)$

Learning Outcomes:

1. Learn to use the properties of inverse Laplace transforms
2. Will able to apply convolution theorem
3. Will able to use the Heaviside's expansion

Web Resources:

3. **Online web notes -Inverse Laplace transform:**
[https://math.libretexts.org/Bookshelves/Differential_Equations/Introduction_to_Partial_Differential_Equations_\(Herman\)/09%3A_Transform_Techniques_in_Physics/9.09%3A_The_Convolution_Theorem](https://math.libretexts.org/Bookshelves/Differential_Equations/Introduction_to_Partial_Differential_Equations_(Herman)/09%3A_Transform_Techniques_in_Physics/9.09%3A_The_Convolution_Theorem)
4. YouTube videos : <https://www.youtube.com/watch?v=Cn8KwBjrdcA>

Text Books:

1. Vashistha A.R & Dr. Guptha R.k,2017, *Integral transform*, 37th edition, Krishna prakashan media Pvt..Ltd.,meeru.

References:

1. Goyal J.K & Guptha K.P, 2011, *Laplace and Fourier transform*, 24 th edition, PragathiPrakashan.
2. Raisinghanian .M.D, 1995, *integral transforms*, 2nd edition S-Chand & co.

Model Paper

Course Code: 23MAMAL233

Time: 3Hrs

Offered to: B.Sc.Hons.Mathematics

Title of the Course: Laplace Transforms

MAX MARKS: 70

SECTION – A

Answer the following.

5 x 4 = 20 M

1.(a) Find $L(7e^{2t} + 9e^{-2t} + 5\cos t + 7t^3 + 5\sin 3t + 2)$

(CO1,K1)

(OR)

1.(b) Find the Laplace transform of the function $F(t) = 4, 0 < t < 1$
 $3, t > 1.$

(CO1,K1)

2. (a) If $L[F(t)] = \frac{p^2 - p + 1}{(2p + 1)^2(p - 1)}$ then show that $L\{F(2t)\} = \frac{p^2 - 2p + 4}{4(p + 1)^2(p - 2)}$ by applying change of scale property

(CO2,K2)

(OR)

2. (b) State and prove Second shifting theorem

(CO2,K2)

3. (a) Find $L\left\{\int_0^t \frac{e^t \sin t}{t} dt\right\}$

(CO3,K2)

(OR)

3. (b) Find the Laplace transform of e^{at} using the theorem on transforms of derivatives

(CO3,K2)

4.(a) Find $L^{-1}\left(\frac{3p - 4}{p^2 - 4p + 8}\right)$

(CO4,K3)

(OR)

4.(b) find the inverse Laplace transform of $\frac{3}{p^2 - 3} + \frac{3p + 2}{p^3} - \frac{3p - 27}{p^2 + 9} + \frac{6 - 30\sqrt{p}}{p^4}$

(CO4,K3)

5.(a) Find $L^{-1}\left(\frac{1}{p} \log\left(\frac{p+2}{P+1}\right)\right)$ by division by 'p'

(CO5,K3)

(OR)

5.(b) Find $L^{-1}\left(\frac{P + 3}{(p^2 + 6p + 13)^2}\right)$

(CO5,K3)

SECTION-B

Answer the following

10x5=50M

6.(a) Find $L(\sin t \cos t)$

(CO1,K2)

(OR)

6.(b) using expansion $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$ Show that $L(\sin \sqrt{t}) = \frac{\sqrt{\pi}}{2p^{3/2}} e^{-1/4p}$

(CO1,K3)

7.(a) State and prove Initial value theorem

(CO2,K3)

(OR)

7.(b)(i) If $L(F(t)) = \frac{9p^2 - 12p + 15}{(p-1)^3}$ then find $L(F(3t))$ using change of scale property (CO2,K3)

(ii) Find $L(G(t))$ where $G(t) = \begin{cases} \cos(t - \frac{\pi}{3}), t > \frac{\pi}{3} \\ 0, t < \frac{\pi}{3} \end{cases}$ (CO2,K3)

8.(a) Find $L((t^2 - 3t + 2)\sin 3t)$ (CO3,K4)

(OR)

(b)(i) Find the Laplace transform of $L\left(\frac{\sin 3t \cos t}{t}\right)$ (CO3,K4)

(ii) Find the Laplace transform of $\left(\frac{\cos at - \cos bt}{t}\right)$ (CO3,K4)

9.(a) Find the inverse Laplace transforms of $\frac{3p+1}{p^2 - 2p - 3}$ by partial fractions (CO4,K4)

(OR)

9. (b) (i) Find $L^{-1}\left(\frac{e^{4-3p}}{(p+4)^{5/2}}\right)$ (CO4,K4)

(ii) Find the inverse Laplace of $\frac{e^{-\pi p}(p+1)}{p^2 + p + 1}$ (CO4,K4)

10.(a) Using Heaviside's expansion formula, Find $L^{-1}\left(\frac{3p+1}{(p-1)(p^2+1)}\right)$ (CO5,K4)

(OR)

10.(b) Using convolution theorem, $L^{-1}\left(\frac{1}{p(p+1)(p+2)}\right)$ (CO5,K4)

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Course Code				23MAMAL234			
Title of the Course				Special functions			
Offered to: (Programme/s)				B.Sc Hons (Mathematics)			
L	5	T	1	P	0	C	4
Year of Introduction:		2024-25		Semester:		3	
Course Category:		MAJOR		Course Relates to:		GLOBAL	
Year of Revision:				Percentage:		NA	
Type of the Course:				SKILL ENHANCED			
Crosscutting Issues of the Course :				GENDER			
Pre-requisites, if any				Basics of Special functions			

Legend:

Offered to: Name of the Program (B.Sc. Hons Mathematics)

Category: Major

Course Relates to: Global

Type of the Course: Skill Enhancement Course

Crosscutting Issues of the Course: Beta and Gamma functions

L: Lecture; T: Tutorial; P: Practicum/Practical/Project; C: Credits

Course Description:

The properties of special functions like Gauss hypergeometric, Legendre functions with their integral representations. Understand the concept of Bessel's function, Hermite function etc, with its properties like recurrence relations, orthogonal properties, generating functions.

Course Aims and Objectives:

S.NO	COURSE OBJECTIVES
1	Acquire the information about Beta and Gamma functions, and evaluate it in various Problems.
2	Derive Rodrigue's formula, generating function, recurrence relations and orthogonal Property of Laguerre polynomials and use them in various applications
3	Solve Hermite equation and write the Hermite Polynomial of order 'n' also find the generating function and orthogonal properties of Hermite polynomials
4	Solve Legendre equation and write the Legendre equation of first kind, also find the generating function and orthogonal properties of Legendre Polynomials
5	Solve Bessel's equation and write the Bessel's equation of first kind also find the Generating function of Bessel's function.

Course Outcomes

At the end of the course, the student will be able to...

CO NO	COURSE OUTCOME	BTL	PO	PSO
CO1	Understand concepts of Beta and Gamma functions	K3	1	1
CO2	Orthogonal Property of Laguerre polynomials and use them in various applications.	K1	2	2
CO3	Explain the orthogonal properties of Hermite polynomials	K2	1	1
CO4	Explain orthogonal properties of Legendre Polynomials	K2	1	1
CO5	Find generating function of Bessel's function.	K4	2	2

For BTL: K1: Remember; K2: Understand; K3: Apply; K4: Analyze; K5: Evaluate; K6: Create

CO NO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PSO1	PSO2
CO1	2								2
CO2		2							3
CO3	3								2
CO4	3								2
CO5		2							2

Course Structure:

UNIT-I:

UNIT – I: BETA AND GAMMA FUNCTIONS, CHEBYSHEV POLYNOMIALS (15hrs)

- 1.1 - Euler's Integrals-Beta and Gamma Functions, Elementary properties of Gamma Functions,
- 1.2 - Transformation of Gamma Functions, Another form of Beta Function,
- 1.3 - Relation between Beta and Gamma Functions.
- 1.4 - Chebyshev polynomials, orthogonal properties of Chebyshev polynomials
- 1.5 - Recurrence relations, generating functions for Chebyshev polynomials

Description: This course delves into the Beta and Gamma functions, which are fundamental in various areas of mathematics and applied sciences. The Beta and Gamma functions are integral transforms that play a critical role in complex analysis, probability theory, and mathematical physics.

Examples/Applications/Case Studies:

- Generalized Functions: The Beta and Gamma functions are often used together in problems involving integrals that appear in generalized functions and in the study of hypergeometric functions.
- Numerical Methods: Both functions are utilized in numerical methods for evaluating integrals and solving differential equations that cannot be solved analytically.

Exercises:

7. P.T $\Gamma\left(\frac{3}{2} - x\right) \Gamma\left(\frac{3}{2} + x\right) = \left(\frac{1}{4} - x^2\right) \pi \sec \pi x$ where $-1 < 2x < 1$
8. Find the values of $\Gamma\left(\frac{1}{2}\right)$, $\Gamma(0)$, $\Gamma(1)$
9. Derive the symmetric property of Beta function.

Learning Outcomes:

1. **Conceptual Understanding:**
 - Develop a solid grasp of the definitions, properties, and relationships of the Beta and Gamma functions.
2. **Computational Skills:**
 - Gain proficiency in calculating values and integrals involving Beta and Gamma functions.
 - Use recurrence relations and special cases to simplify computations.
3. **Application Knowledge:**
 - Apply these functions in various mathematical contexts such as integrals, series, and probability theory.
 - Understand their role in statistical distributions and their connection to factorials.
4. **Analytical Techniques:**
 - Learn to manipulate these functions to solve complex problems, particularly in calculus and complex analysis.
5. **Theoretical Insight:**
 - Explore how Beta and Gamma functions link different mathematical areas and their importance in theoretical development.

Web Resources:

5. **Online Math Notes –**
6. <https://web.mst.edu/~lmhall/SPFNS/spfns.pdf>

UNIT –II: LAGUERRE POLYNOMIALS

- 2.1 - Laguerre's differential equation
- 2.2 - Laguerre polynomials
- 2.3 - Generating function

- 2.4 - Other forms for Laguerre polynomials
- 2.5 - Rodrigue's formula
- 2.6 - To find first few Laguerre polynomials
- 2.7 - Orthogonal properties for Laguerre polynomials
- 2.8 - Recurrence formula for Laguerre polynomials.

Description Laguerre polynomials are a sequence of orthogonal polynomials that arise in various areas of mathematical analysis and physics, particularly in the study of quantum mechanics and differential equations. They are named after the French mathematician Edmond Laguerre..

Examples/Applications/Case Studies:

Angular Momentum: They also appear in the study of angular momentum in quantum mechanics, particularly in problems involving spherical coordinates. Find the number of Proper subgroups of the given set.

Differential Equations: They are solutions to the Laguerre differential equation, which is used in various physical models and boundary value problems.

Exercises:

- 8. Compute $L_0(x)$, $L_1(x)$, $L_2(x)$ explicitly.
- 9. Prove that $L_n(0) = 1$.

Learning Outcomes:

- 10. Understanding of Definitions and Basic Properties
- 11. Application in Differential Equations
- 12. Usage in Quantum Mechanics

Web Resources:

- 5. <https://web.mst.edu/~lmhall/SPFNS/spfns.pdf>

UNIT-III: HERMITE POLYNOMIALS

- 3.1. Hermite Differential Equations, Solution of Hermite Equation, Hermite polynomials,
- 3.2. Generating function for Hermite polynomials.
- 3.3. Other forms for Hermite Polynomials, Rodrigues formula for Hermite Polynomials, to find first few hermite Polynomials.
- 3.4. Orthogonal properties of Hermite Polynomials, Recurrence formulae for Hermite Polynomials.

Description: Hermite polynomials are a family of orthogonal polynomials that arise in probability theory, combinatorics, and the solution of differential equations. They are particularly important in quantum mechanics and statistical physics

Examples/Applications/Case Studies:

Quantum Mechanics

- **Quantum Harmonic Oscillator:** Hermite polynomials are fundamental in solving the Schrödinger equation for the quantum harmonic oscillator. The eigenfunctions of the harmonic oscillator are expressed in terms of Hermite polynomials, which describe the probability distributions of the oscillator's energy levels
- **Normal Distribution:** Hermite polynomials are used in the expansion of the normal distribution.

Orthogonal Polynomials: Hermite polynomials are used in approximation theory for constructing orthogonal polynomial expansions, which can approximate functions over the real line with a known weight function

Exercises:

- 6. Verify the first few Hermite polynomials using their recurrence relation:

$$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$$
- 7. Find $H_2(x)$, $H_3(x)$,

Learning Outcomes:

- The probabilistic and functional forms of Hermite polynomials are deeply connected with Gaussian functions, particularly in how they serve as eigenfunctions of certain differential operators.

Mastering Hermite polynomials involves understanding their algebraic properties, their role in mathematical physics, and their practical applications across various scientific fields

Web Resources:

- 7. **Online Math Notes –**
- 1. <http://www.physics.wm.edu/~finn/home/MathPhysics.pdf>

UNIT-IV: LEGENDRE'S POLYNOMIALS

- 4.1. Definition, Solution of Legendre's equation, Legendre polynomial of degree n,
- 4.2. Generating function of Legendre polynomials.
- 4.3. Definition of $P_n(x)$ and $Q_n(x)$, General solution of Legendre's Equation (derivations not required) to show that $P_n(x)$ is the coefficient of h^n , in the expansion of $(1 - 2xh + h^2)^{-1/2}$.
- 4.4. Orthogonal properties of Legendre's polynomials, Recurrence formulas for Legendre's Polynomials.

Description: Legendre polynomials are a class of orthogonal polynomials that arise in various areas of mathematics and physics. They are particularly significant in solving problems with spherical symmetry, such as in potential theory and quantum mechanics

Examples/Applications/Case Studies:

5. Spherical Harmonics Expansion.
6. **Hydrogen Atom**
7. Numerical Integration

Exercises:

1. Verify the recurrence relation for Legendre polynomials:
2. Derive the generating function for Legendre polynomials:.

Learning Outcomes:

7. Understand the concept of Electrostatics and Potential Theory
8. Understand the concept of **Geophysics and Astronomy**:

Web Resources:

2. **Online Math Notes** –
https://www.math.tamu.edu/~fnarc/psfiles/special_fun.pdf

UNIT-V: BESSEL'S EQUATION

(15 hrs)

- 5.1. Definition, Solution of Bessel's equation, Bessel's function of the first kind of order n,
- 5.2. Bessel's function of the second kind of order n.
- 5.3. Integration of Bessel's equation in series form=0,
- 5.4. Definition of $J_n(x)$, recurrence formulae for $J_n(x)$.
- 5.5. Generating function for $J_n(x)$, orthogonally of Bessel function

Description:

Bessel functions are a family of solutions to Bessel's differential equation, which appears in many problems of mathematical physics, particularly those involving cylindrical symmetry. They play a crucial role in fields such as wave propagation, heat conduction, and electromagnetic theory.

Examples/Applications/Case Studies:

6. **Wave Propagation**
7. Heat Conduction

Exercises:

5. Derive $J_{\frac{1}{2}}(x)$, $J_{-\frac{1}{2}}(x)$.
6. Prove that $2J'_n(x) = J_{n-1}(x) - J_{n+1}(x)$.

Learning Outcomes:

7. Define and understand the properties of Bessel's equation
8. Solve problems involving the Bessel's equation

Web Resources:

3. **Online web notes:** <https://nitkkr.ac.in/docs/18-%20Series%20Solution%20and%20Special%20Functions.pdf>

Text Books :

1. J.N. Sharma and Dr.R.K. Gupta, Special functions, Krishna Prakashan Mandir, Meerut.

Reference Books:

1. Shanti Narayan and Dr.P.K.Mittal, Integral Calculus, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.

Model Paper

Course Code: 23MAMAL234

Time: 3Hrs

Offered to: B.Sc HONS (MATHEMATICS)

Title of the Course: SPECIAL FUNCTIONS

MAX MARKS: 70

SECTION – A

Answer any FIVE Questions

5x4=20M

1. (a). Evaluate $\int_0^{\infty} x^2 e^{-x^2} dx$ (CO1,K1)

(OR)

- (b). P.T $\Gamma\left(\frac{3}{2} - x\right) \Gamma\left(\frac{3}{2} + x\right) = \left(\frac{1}{4} - x^2\right) \pi \sec \pi x$ where $-1 < 2x < 1$ (CO1,K1)

- 2 (a). Prove that $L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$. (CO2,K2)

(OR)

- (b). Prove that $L_n(0) = 1$. (CO2,K2)

- 3 (a). Prove that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$ (CO3,K3)

(OR)

- (b). Prove that $H_n^1(x) = 2nH_{n-1}(x)$ (CO3,K3)

- 4 (a). Prove that $P_n(x) (-x) = (-1)^n P_n(x)$ (CO4,K3)

(OR)

- (b). Prove that $(1 - x^2)P_n^1(-x) = (n + 1)(xP_n(x) - P_{n+1}(x))$ (CO4,K3)

- 5 (a) Prove that $xJ_n^1(x) = nJ_n(x) - xJ_{n+1}(x)$ (CO5,K3)

(OR)

- (b) State and prove Generating function for $J_n(x)$. (CO5,K2)

SECTION – B

Answer all Questions

5x10=50M

6. (a). Derive the relation between Beta and Gamma functions. (CO1,K1)

(OR)

- (b). When n is a positive integer prove that $2^n \Gamma\left(n + \frac{1}{2}\right) = 1.3.5 \dots (2n-1) \sqrt{\pi}$. (CO1,K1)

7. (a) State and Prove Orthogonal properties of Laguerre polynomials.

(CO2,K2)

(OR)

- (b). Prove that $(n+1) L_{n+1}(x) = (2n+1-x) L_n(x) - n L_{n-1}(x)$. (CO2,K2)

8. (a) State and Prove Generating function for Hermite Polynomials (CO3,K3)

(OR)

- (b). Prove that $H_n(x) = 2^n \left[\exp\left(-\frac{1}{4} \frac{d^2}{dx^2}\right) x^n \right]$. (CO3,K3)

9. (a) Prove that $(2n+1) x P_n(x) = (n+1) P_{n+1}(x) + n P_{n-1}(x)$. (CO4,K3)

(OR)

- (b). State and prove Rodrigues formula for Legendre's Equation. (CO4,K3)

10. (a) Prove that $2J_n'(x) = J_{n-1}(x) - J_{n+1}(x)$. (CO5,K3)

(OR)

- (b). Prove that $\sqrt{\left(\frac{\pi x}{2}\right)} J_{\frac{3}{2}}(x) = \frac{1}{x} \sin x - \cos x$. (CO5,K3).

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Course Code				23MAMAL235			
Title of the Course				Linear Algebra & Matrices			
Offered to: (Programme/s)				B.Sc Hons (Data Science)			
L	5	T	0	P	0	C	4
Year of Introduction:		2024-25		Semester:			3
Course Category:		MAJOR		Course Relates to:		GLOBAL	
Year of Revision:				Percentage:		NA	
Type of the Course:				SKILL ENHANCED			
Crosscutting Issues of the Course :				GENDER			
Pre-requisites, if any				Basics of Algebraic Structures			

Legend:

Offered to: Name of the Program (B.Sc. Hons **Data Science**)

Category: Major

Course Relates to: Global

Type of the Course: Skill development

Crosscutting Issues of the Course: Vector Space, Basis and dimension, Linear Transformation, matrices.

L: Lecture; T: Tutorial; P: Practicum/Practical/Project; C: Credits

Course Description:

Linear algebra is a branch of mathematics that studies systems of linear equations and properties of matrices. This course covers vector spaces, matrices, system of linear equations, determinants, eigen values and eigen vectors, and linear transformations.

Course Aims and Objectives:

S.N O	COURSE OBJECTIVES
1	Understand the concept of Vector Space and study its properties. Identifying Linear independence and independence of vectors.
2	Apply knowledge to identify Basis of a Vector Space and dimension of it.
3	Understand the Vector Space Homomorphism, linear transformation and its properties, facilitating a deeper understanding of range and null space of a linear transformation.
4	Apply knowledge to determine echelon form of a matrix, rank of a matrix and reduction of normal form.
5	Apply knowledge to identify and characterize eigen values and eigen vectors.

Course Outcomes

At the end of the course, the student will be able to...

CO NO	COURSE OUTCOME	BTL	PO	PSO
CO1	Explain concepts of vector space and its properties.	K2	1	1
CO2	Compute basis and dimension of vector space.	K3	2	2
CO3	Identify range and null space of a linear transformation.	K2	2	1
CO4	Compute rank and inverse of a matrix.	K3	2	2
CO5	Compute eigen values and eigen vectors of a matrix.	K3	2	2

For BTL: K1: Remember; K2: Understand; K3: Apply; K4: Analyze; K5: Evaluate; K6: Create

CO-PO MATRIX									
CO NO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PSO1	PSO2
CO1	3								3
CO2		3							2
CO3	2								2
CO4		3							3
CO5		3							2

Course Structure:

UNIT-I: Vector Spaces

Vector space definition – general properties of Vector space, Subspace definition – related problems, Linear sum of two subspaces, linear combination of vectors and linear span of a set – related problems, Linear dependence of vectors definition - related problems, Linear independence of vectors definition -related problems.

Description: This unit familiarizes the students, the concepts of Vector Space, Subspace and linearly dependent & independent vectors.

Examples/Applications/Case Studies:

1. Explain the definition of a Vector space with an example.
2. Examine the following vectors are linearly dependent or linearly independent $(1,2,0)$, $(0,3,1)$, $(-1,0,1)$.

Exercises:

1. Prove that the set $\{(a_1, a_2): a_1, a_2 \in \mathbb{R}\}$ is a vector space over \mathbb{R} w.r.t. the operations addition and scalar multiplication defined as $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2)$ and $c(a_1, a_2) = (ca_1, ca_2)$, where $(a_1, a_2), (b_1, b_2) \in V$ and $c \in \mathbb{R}$.
2. Prove that $W = \{(a_1, a_2, 0): a_1, a_2 \in F\}$ is a subspace of $V_3(F)$, where F is a field.

Learning Outcomes:

1. Understanding the properties of vector space, external composition and internal composition.
2. Practicing problems on different vector spaces enhances problem-solving abilities and logical reasoning.

Web Resources:

1. Online Math Notes - Vector Spaces:

https://sgfin.github.io/files/notes/NYU_Optimization_2017.pdf

UNIT-II: Basis and Dimension

Basis of a vector space – definition. Basis existence, Basis extension theorems, Dimension of a vector space - related problems, Dimension of a subspace theorems- related problems, Quotient space – definition.

Description: A subset S of a vector space $V(F)$ is said to be a basis of $V(F)$, if (i) S is a linearly independent set.(ii) S spans V , $L(S)=V$. In mathematics the dimension of a vector space V is the cardinality (i.e., the number of vectors) of a basis of V over its base field.

Examples/Applications/Case Studies:

1. Show that the vectors $(1,2,1)$, $(2,1,0)$, $(1,-1,2)$ form a basis for \mathbb{R}^3 .
2. Define the quotient space.

Exercises:

1. Find the dimension of given subspace.
2. Prove that the set $\{(1,0,0)$, $(1,1,0)$, $(1,1,1)$, $(0,1,0)\}$ spans the vector space $\mathbb{R}^3(\mathbb{R})$.

Learning Outcomes:

1. Understand the concept of Basis and dimension of a vector space.
2. Determine whether the given set of vectors forms a basis or not.

Web Resources:

1. Basis and Dimension problems and solutions

<https://people.tamu.edu/~vvorobets//MATH304-2011C/Lect2-06web.pdf>

UNIT-III: Linear Transformations

Vector space homomorphism – definitions, Linear transformation, Properties of L.T.,

Sum of linear transformations, scalar multiplication of L.T., product of linear transformations, Algebra of linear operators - related problems, Range & Null space of a L.T. – Definitions, related problems, Rank nullity theorem – related problems.

Description: A linear transformation is a function from one vector space to another that respects the underlying (linear) structure of each vector space. If the two vector spaces are same then that linear transformation is known as a linear operator or map.

Examples/Applications/Case Studies:

1. Define range and null space of a linear transformation.
2. Show that the given mapping is a linear transformation.

Exercises:

1. Describe explicitly the linear transformation using given conditions.
2. Test whether the given mapping is a linear transformation or not.

Learning Outcomes:

1. Determine a linear transformation from given conditions.
2. Determine range of a linear transformation.

Web Resources:

1. **Online Math Notes –Linear Transformations:**
<https://mandal.ku.edu/math290/m290NotesChSIX.pdf>

UNIT-IV: MATRICES – I

Fundamentals of Matrices, Elementary matrix operations & elementary matrices, Rank of a matrix – definition, related problems, Echelon form of a matrix, reduction to normal form, Inverse of a matrix – related problems only.

Description: Elementary matrix operations are fundamental tools in linear algebra used for solving systems of linear equations, finding matrix inverses.

Examples/Applications/Case Studies:

8. Using elementary row operations to find the rank of a given matrix.
9. Find inverse of the matrix.

Exercises:

5. Solve upper triangular and lower triangular matrices and also find its rank.
6. Reduce the given matrix into normal form.

Learning Outcomes:

9. Use elementary transformations, find inverse of a given matrix.
10. Reduce the given matrix into echelon form.

Web Resources:

Online Math Notes – Matrices – I :

7. chrome-extension://efaidnbmnnnibpcajpcglclefindmkaj/https://ocw.mit.edu/courses/18-06sc-linear-algebra-fall-2011/1999c9f4accdbef05571a1014438f8dd_MIT18_06SCF11_Ses2.8sum.pdf

UNIT-V: MATRICES – II

System of linear equations – homogeneous linear equations – related problems, System of linear equations – non homogeneous linear equations – related problems, Eigen values & Eigen vectors of a matrix – definitions & related problems, Cayley - Hamilton theorem statement only, related problems.

Description: Eigen values and eigenvectors are fundamental concepts in linear algebra with applications across many fields, including physics, engineering, computer science, and data analysis.

Examples/Applications/Case Studies:

8. Solve the given system of homogeneous linear equations.
9. Solve the given system of non-homogeneous linear equations.
10. Find the Eigen values and Eigen vectors.

Exercises:

9. Investigate the Eigen values from the given system of equations.
10. Solve the polynomial equation by using Cayley Hamilton theorem.

Learning Outcomes:

9. Recognize and use equivalent forms to identify matrices and solve linear systems
10. Find the characteristic polynomial of a matrix.

Web Resources:**5. Online web notes: Matrices – II .**

chrome-extension://efaidnbmnnnibpcajpcglclefindmkaj/https://ocw.mit.edu/courses/18-06sc-linear-algebra-fall-2011/1999c9f4accdbef05571a1014438f8dd_MIT18_06SCF11_Ses2.8sum.pdf

6. YouTube videos by AVM Math hub (Student connect):

<https://www.youtube.com/watch?v=5bsaKvzTyyo&t=905s>

Text Books:

2. Venkateswara Rao V & Krishna Murthy N. (2006). *A textbook of Mathematics for B.A/B.Sc - Vo.l-III* (2nd Edition). S – Chand.

Reference Books:

3. Dr Anjaneyulu A. (2006). *A textbook of Mathematics for B.A / B.Sc - Vo.l- III* (3rd Edition). Deepthi Publications.
4. Sharma J.N & Vasistha A. R. (2010). *A text book of Linear Algebra (42nd Edition)*. Krishna Prakashan Mandir.

Model Paper

Course Code: 23MAMAL235

Time: 3Hrs

Offered to: B.Sc HONS (DATA SCIENCE)

Title of the Course: LINEAR ALGEBRA & MATRICES

MAX MARKS: 70

SECTION – A

I Answer the following

5 x 4 = 20M

1. (a) The set W of ordered triads $(x, y, 0)$, where $x, y \in F$ is a subspace of $V_3(F)$. **(CO1,K1)**

(OR)

(b) Show that the system of vectors $(1, 3, 2), (1, -7, -8), (2, 1, -1)$ of $V_3(R)$ is L.D. **(CO1,K1)**

2. (a) Define Basis of a vector space. Give an example. **(CO2,K2)**

(OR)

(b) Show that the vectors $\{(1, 2, 1), (2, 1, 0), (1, -1, 2)\}$ forms a basis of R^3 **(CO2,K2)**

3. (a) Describe explicitly the linear transformation $T: R^2 \rightarrow R^2$ such that $T(2, 3) = (4, 5)$ and $T(1, 0) = (0, 0)$. **(CO3,K3)**

(OR)

(b) Define range and null space of a linear transformation. **(CO3,K3)**

4. (a) Find the rank of the matrix $\begin{bmatrix} -1 & 2 & 0 \\ 3 & 7 & 1 \\ 5 & 9 & 3 \end{bmatrix}$. **(CO4,K2)**

(OR)

(b) Reduce the matrix into normal form and find the rank of matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$. **(CO4,K2)**

5. (a) Find the Eigen roots of $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ **(CO5,K3)**

(OR)

(b) State Cayley – Hamilton Theorem. **(CO5,K3)**

SECTION – B

II Answer the following

5 x 10 = 50M

6 (a) Prove that the set V_n of all n-tuples over a field F is a vector space w.r.t. addition of n-tuples and multiplication of n-tuple by a scalar. **(CO1,K3)**

(OR)

(b) Express the vector $\alpha = (1, -2, 5)$ as a linear combination of the vectors

$e_1 = (1, 1, 1), e_2 = (1, 2, 3)$ and $e_3 = (2, -1, 1)$. **(CO1,K3)**

7 (a) State and prove Basis existence theorem. **(CO2,K3)**

(OR)

b) Let W_1 and W_2 be two subspaces of R^4 given by

$W_1 = \{(a, b, c, d) : b - 2c + d = 0\}, W_2 = \{(a, b, c, d) : a = d, b = 2c\}$. Find the basis and dimension of i) W_1 ii) W_2 .

(CO2,K3)

8 (a) State and prove Rank nullity theorem. **(CO3,K3)**

(OR)

(b) Find the null space, range, rank and nullity of the transformation $T: R^2 \rightarrow R^3$ defined by $T(x, y) = (x+y, x-y, y)$. **(CO3,K4)**

9 (a) Find the inverse of the Matrix $\begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$ (CO4,K4)

(OR)

(b) Find the Rank of the Matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ (CO4,K4)

10 (a) Investigate for what values of λ, μ the simultaneous equations $x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$
 i) No solution ii) Unique solution iii) Infinite no. of solutions. (CO5,K3)

(OR)

(b) If $A = \begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix}$ verify Cayley – Hamilton theorem and hence find A^{-1} . (CO5,K3)



SRI DURGA MALLESWARA SIDDHARTHA MAHILA KALASALA, VIJAYAWADA - 10.

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Course Code				23MAMIL231			
Title of the Course				Laplace transforms			
Offered to: (Programme/s)				Bsc.Hons(Physics)			
L	4	T	0	P	0	C	4
Year of Introduction:		2024-25		Semester:			3
Course Category:		Minor		Course Relates to:		Local	
Year of Revision:				Percentage:		NA	
Type of the Course:				Skill Enhanced			
Crosscutting Issues of the Course :							
Pre-requisites, if any							

Legend:

Offered to: Name of the Program (B.Com Hons/ B.A Hons/B.Sc. Hons/B.C.A Hons/ B.B.A Hons)

Category: Major/Minor/MDC/SDC/ENG/TEL/VAC

Course Relates to:Local, Regional, National, Global

Type of the Course:Employability/ Entrepreneurship/ Skill development

Crosscutting Issues of the Course: Gender, Environment and Sustainability, Human Values and Professional Ethics

L: Lecture; T: Tutorial; P: Practicum/Practical/Project; C: Credits

Course Description:

An overview of the course content and objectives.

Laplace transform is a fundamental tool in integral calculus.it is used to solve various types of differential equations, difference equations, integral equations etc. which arise naturally in engineering and basic sciences

Course Aims and Objectives:

S.NO	COURSE OBJECTIVES
1	Knowledge in fundamental concepts of Laplace transforms of a function
2	Ability to develop necessary skills to recognize the properties of Laplace transforms their applications
3	Understand Laplace transforms of various functions
4	Ability to understand the basic concepts of inverse transforms of a functions
5	Competence in concepts of convolution theorem, Heaviside expansion and their applications

Course Outcomes

At the end of the course, the student will be able to...

CO NO	COURSE OUTCOME	BT L	P O	PS O
CO1	Understand the definition and properties of Laplace transformations	K1	1	2

CO2	Get an idea about first and second shifting theorems and change of scale property	K2	1	2
CO3	Analyse the Laplace transform of various functions	K4	2	2
CO4	Know the reverse transformation of Laplace and its properties	K2	2	2
CO5	Get the knowledge of application of convolution theorem	K3	2	2

For BTL: K1: Remember; K2: Understand; K3: Apply; K4: Analyze; K5: Evaluate; K6: Create

CO-PO MATRIX									
CO NO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PSO1	PSO2
CO1	3								2
CO2	2								2
CO3		2							2
CO4		2							2
CO5		2							2

Use the codes 3, 2, 1 for High, Moderate and Low correlation Between CO-PO-PSO respectively

Course Structure:

Unit-1 Laplace transforms-1

15 hrs

Definition of Laplace transform- linearity property- piecewise continuous functions-Existence of Laplace transforms- functions of exponential order and of class A, First shifting theorem.

Description: This unit familiarizes the students, the concept of Laplace transforms. The Laplace transform is used to solve differential equations.

Examples/Applications/Case Studies:

3. Explain how to apply Laplace transforms for given functions.
4. Extend the different properties of Laplace transforms can be applied or not
5. Combining some of these simple Laplace transforms with the properties of the Laplace transforms

Exercises:

3. Find $L(\sin 2t \cos 3t)$
4. Find the Laplace transform of the function $F(t) = 4, 0 < t < 1$
 $3, t > 1.$
5. Evaluate $L(\sin t \cos t)$

Learning Outcomes:

3. Learn the application of Laplace transform in engineering analysis.
4. Understanding the properties of Laplace transforms reinforces foundational concepts in integral transforms
5. Learn the required conditions for transforming variable or variables in functions by the Laplace transform

Web Resources:

2. Online Math Notes – Laplace transforms -I:

https://sist.sathyabama.ac.in/sist_coursematerial/uploads/SMT1401.pdf

3. Youtube Videos :

https://www.youtube.com/watch?v=5hPD7CF0_54

Unit-2: Laplace Transforms-II

Second Shifting Theorem, Change of Scale Property, Laplace transform of the derivative of $f(t)$, Initial value theorem and Final value theorem.

Description: The second shift theorem is similar to the first except that, in this case. It is the time variable that is shifted not the t - variable

Examples/Applications/Case Studies:

3. If $L(F(t)) = \frac{9p^2 - 12p + 15}{(p-1)^3}$ then find $L(F(3t))$ using change of scale property

4. Evaluate $L(G(t))$ where $G(t) = \begin{cases} \cos(t - \frac{\pi}{3}), t > \frac{\pi}{3} \\ 0, t < \frac{\pi}{3} \end{cases}$

5. State and prove initial value theorem

Exercises:

3. Find the Laplace transform of $\cos at$ using the theorem on transforms of derivatives
4. Find $L(\sin^2(at))$ by using change of scale property

Learning Outcomes:

3. Understand the properties of Laplace transform
4. Determine how to apply the properties to given functions.
5. Will able to solve the change of scale, second shifting, transform of derivatives problems

Web Resources:

3. Laplace transform -II

<https://byjus.com/maths/Laplace-transform/>

4. YouTube videos :

<https://www.youtube.com/watch?v=C0GPZZ15Shk>

Unit-3:Laplace Transforms-III

Laplace transform of Integrals and related problems– Multiplication by t , Multiplication by t^n and related problems- division by t and related problems- Evaluation of integrals by Laplace transforms

Description: The Laplace transform's key property is that it converts differentiation and integration in the time domain into multiplication and division in the Laplace domain. The Laplace transform is an integral transform that can be used to evaluate integrals, it converts a function of real variable into a function of a complex variable.

Examples/Applications/Case Studies:

4. Find $L\left(\frac{e^{-at}-e^{-bt}}{t}\right)$
5. Evaluate $L(t \cos 3t)$
6. Show that $\int_0^{\infty} t e^{-3t} \sin t \, dt = \frac{3}{50}$

Exercises:

4. Find the Laplace transform of $\frac{1-\cos t}{t^2}$
5. Find $L((t^2-3t+2)\sin 3t)$
6. Evaluate $\int_0^{\infty} t e^{-3t} \, dt$

Learning Outcomes:

3. Will able to know that how to apply the multiplication by 't' and division by 't'
4. Will able to understand the concepts of division by t and evaluation of integrals of Laplace transforms.
5. Able to solve the problems of Laplace transforms

Web Resources:

2. **Online Math Notes- Laplace transform-III:** https://mathalino.com/reviewer/advance-engineering-mathematics/evaluation-integrals#google_vignette
3. **YouTube videos :** <https://www.youtube.com/watch?v=sIGB8nKddUw>

Unit -4: Inverse Laplace transforms –I

Definition of inverse Laplace transform – linearity property - First shifting Theorem-Second Shifting Theorem- Change of Scale Property- use of partial fractions – examples

Description: The inverse Laplace transform is used to find the original function from its Laplace transform. It's a powerful tool for solving non homogeneous linear differential equations which are equations where the solution to the derivative is not zero

Examples/Applications/Case Studies:

1. Find the inverse Laplace transform of $\frac{2p-5}{p^2-9}$
2. Evaluate inverse Laplace transform of $\left(\frac{p+1}{p^2+6p+25}\right)$

Exercises:

5. Find the inverse Laplace transform of $\frac{e^{-3p}}{(p-2)^2}$
6. Find the inverse Laplace transforms of $\frac{3p+1}{p^2-2p-3}$ by partial fractions

Learning Outcomes:

11. Understand the concept of inverse Laplace transforms.
12. Learn to use inverse Laplace transform to solve problems.

Web Resources:

8. **Online Math Notes – inverse Laplace transform –I:** <https://www.vedantu.com/maths/inverse-Laplace-transform>
9. **YouTube videos :** <https://www.youtube.com/watch?v=Y8GXpS31CGI>

Unit -5: Inverse Laplace transforms –II

Inverse Laplace transforms of derivatives- inverse Laplace transforms of Integrals- multiplication by powers of 'p'-Division by powers of 'p'-convolution Definition- Convolution theorem –Proof and Applications- Heaviside's expansion theorem and its Applications

Description: The convolution theorem for Laplace transforms states that taking the convolution of two functions and then taking the Laplace transform is the same as taking the Laplace transform of each function separately and then multiplying the two Laplace transform together

Examples/Applications/Case Studies:

3. Find $L^{-1} \left(\frac{1}{p} \log \left(\frac{p+2}{P+1} \right) \right)$ by division by 'p'
4. Find $L^{-1} \left(\frac{P+3}{(p^2 + 6p + 13)^2} \right)$

Exercises:

7. Using Heaviside's expansion formula, Find $L^{-1} \left(\frac{3p+1}{(p-1)(p^2+1)} \right)$
8. Using convolution theorem, $L^{-1} \left(\frac{1}{p(p+1)(p+2)} \right)$

Learning Outcomes:

4. Learn to use the properties of inverse Laplace transforms
5. Will able to apply convolution theorem
6. Will able to use the Heaviside's expansion

Web Resources:

7. **Online web notes -Inverse Laplace transform:** [https://math.libretexts.org/Bookshelves/Differential_Equations/Introduction_to_Partial_Differential_Equations_\(Herman\)/09%3A_Transform_Techniques_in_Physics/9.09%3A_The_Convolution_Theorem](https://math.libretexts.org/Bookshelves/Differential_Equations/Introduction_to_Partial_Differential_Equations_(Herman)/09%3A_Transform_Techniques_in_Physics/9.09%3A_The_Convolution_Theorem)
8. YouTube videos : <https://www.youtube.com/watch?v=Cn8KwBjrdcA>

Text Books:

1. Vashistha A.R & Dr. Guptha R.k,2017, *Integral transform* ,37 th edition Krishna prakashan media Pvt..Ltd.,meeru.

References:

- 1..Goyal .J.K & Guptha K.P, 2011, *Laplace and Fourier transform* , 24th edition ,PragathiPrakashan.
2. Raisinghanian .M.D, 1995, *Integral transforms*, 2nd edition S-Chand & co.

Course Code: 23MAMIL231

Time: 3Hrs

Offered to: B.Sc.Hons.Physics

Title of the Course: Laplace Transforms

MAX MARKS: 70

SECTION – A

Answer the following.

5 x 4 = 20 M

1.(a) Find $L(7e^{2t} + 9e^{-2t} + 5\cos t + 7t^3 + 5\sin 3t + 2)$

(CO1,K1)

(OR)

1.(b) Find the Laplace transform of the function $F(t) = 4, 0 < t < 1$

$3, t > 1.$

(CO1,K1)

2. (a) If $L[F(t)] = \frac{p^2 - p + 1}{(2p + 1)^2(p - 1)}$ then show that $L\{F(2t)\} = \frac{p^2 - 2p + 4}{4(p + 1)^2(p - 2)}$ by applying change of scale property

(CO2,K2)

(OR)

2. (b) State and prove Second shifting theorem

(CO2,K2)

3. (a) Find $L\left\{\int_0^t \frac{e^t \sin t}{t} dt\right\}$

(CO3,K2)

(OR)

3. (b) Find the Laplace transform of e^{at} using the theorem on transforms of derivatives

(CO3,K2)

4.(a) Find $L^{-1}\left(\frac{3p - 4}{p^2 - 4p + 8}\right)$

(CO4,K3)

(OR)

4.(b) find the inverse Laplace transform of $\frac{3}{p^2 - 3} + \frac{3p + 2}{p^3} - \frac{3p - 27}{p^2 + 9} + \frac{6 - 30\sqrt{p}}{p^4}$ (CO4,K3)

5.(a) Find $L^{-1}\left(\frac{1}{p} \log\left(\frac{p + 2}{p + 1}\right)\right)$ by division by 'p'

(CO5,K3)

(OR)

5.(b) Find $L^{-1} \left(\frac{P+3}{(p^2+6p+13)^2} \right)$ (CO5,K3)

SECTION-B

Answer the following

10x5=50M

6.(a) Find $L(\sin t \cos t)$ (CO1,K2)

(OR)

6.(b) using expansion $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$ Show that $L(\sin \sqrt{t}) = \frac{\sqrt{\pi}}{2p^{3/2}} e^{-1/4p}$ (CO1,K3)

7.(a) State and prove Initial value theorem

(CO2,K3)

(OR)

7.(b)(i) If $L(F(t)) = \frac{9p^2 - 12p + 15}{(p-1)^3}$ then find $L(F(3t))$ using change of scale property (CO2,K3)

(ii) Find $L(G(t))$ where $G(t) = \begin{cases} \cos(t - \frac{\pi}{3}), t > \frac{\pi}{3} \\ 0, t < \frac{\pi}{3} \end{cases}$ (CO2,K3)

8.(a) Find $L((t^2 - 3t + 2)\sin 3t)$

(CO3,K4)

(OR)

8.(b)(i) Find the Laplace transform of $L\left(\frac{\sin 3t \cos t}{t}\right)$ (CO3,K4)

(ii) Find the Laplace transform of $\left(\frac{\cos at - \cos bt}{t}\right)$ (CO3,K4)

9.(a) Find the inverse Laplace transforms of $\frac{3p+1}{p^2-2p-3}$ by partial fractions (CO4,K4)

(OR)

9.(b) (i) Find $L^{-1}\left(\frac{e^{4-3p}}{(p+4)^{5/2}}\right)$ (ii) Find the inverse Laplace of $\frac{e^{-\pi p}(p+1)}{p^2+p+1}$ (CO4,K4)

10.(a) Using Heaviside's expansion formula, Find $L^{-1} \left(\frac{3p+1}{(p-1)(p^2+1)} \right)$ **(CO5,K4)**

(OR)

10.(b) Using convolution theorem, $L^{-1} \left(\frac{1}{p(p+1)(p+2)} \right)$ **(CO5,K4)**

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Course Code: **MATSET01**

Offered to: B.Sc(MPC, MPCs, MECS, MSCA, MSCS, MCCC)

Domain Subject: **MATHEMATICS**

Semester – **V/VI**

Max. Marks: **100** (IA: 30+ SEE: 70)

Theory Hrs./Week: **6**

Course 6A: NUMERICAL METHODS

Type of the Course: **Skill Enhancement Course** (Elective)

Credits: **5**

I. Course Outcomes: Students at the successful completion of the course will be able to:

CO1: Apply numerical methods with equal intervals, to find out solution of algebraic equations using different methods under different conditions.

CO2: Apply various interpolation methods with unequal intervals and finite difference Concepts.

CO3: Able to Calculate the value of the derivative of a function at some assigned value using different methods.

CO4: Use Trapezoidal rule, Simpson's rule to approximate the value of a definite integral to a Given accuracy.

CO5: Find numerical solutions of ordinary differential equations by using various numerical methods.

II. Syllabus:

(Total Theory Hours: 75)

UNIT-I: FINITE DIFFERENCES & INTERPOLATION WITH EQUAL INTERVALS (15hrs)

1.1. Introduction, Forward differences, backward differences, Central Differences.

1.2. Symbolic relations, nth Differences of Some functions.

1.3. Advancing Difference formula, Differences of Factorial Polynomial, Summation of Series3.

1.4 Newton's formulae for interpolation, Central Difference Interpolation Formulae.

UNIT-II: INTERPOLATION WITH UNEQUAL INTERVALS

(15 hrs)

2.1. Gauss's Forward interpolation formulae, Gauss's backward interpolation formulae,

2.2. Stirling's formula, Bessel's formula.

2.3. Interpolation with unevenly spaced points, divided differences and properties, Newton's divided differences formula.

2.4. Lagrange's interpolation formula, Lagrange's Inverse interpolation formula.

UNIT-III: NUMERICAL DIFFERENTIATION

(15 hrs)

- 3.1. Derivatives using Newton's forward difference formula, Newton's back ward difference formula,
- 3.2. Derivatives using central difference formula, Stirling's interpolation formula,
- 3.3. Newton's divided difference formula, Maximum and minimum values of a tabulated function.

UNIT-IV: NUMERICAL INTEGRATION

(15 hrs)

- 4.1. General quadrature formula one errors, Trapezoidal rule,
- 4.2. Simpson's 1/3- rule, Simpson's 3/8 - rule, and Weddle's rules,
- 4.3. Euler - McLaurin Formula of summation and quadrature, The Euler transformation.

UNIT-V: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS (15 hrs)

- 5.1. Introduction, Solution by Taylor's Series,
- 5.2. Picard's method of successive approximations,
- 5.3. Euler's method, Modified Euler's method, Runge - Kutta methods.

III RECOMMENDED:

2. Gupta and Malik, Calculus of Finite Differences and Numerical Analysis. Krishna Prakashan Mandir, Meerut.

III REFERENCES:

- 1 S.S. Sastry, Introductory Methods of Numerical Analysis, Prentice Hall of India Pvt. Ltd., New Delhi-110001, 2006.
3. P. Kandasamy, K. Thilagavathy, Calculus of Finite Differences and Numerical Analysis. S. Chand & Company, Pvt. Ltd., Ram Nagar, New Delhi-110055.

WEB LINKS:

3. <http://spartan.ac.brocku.ca/~jvr/bik/MATH2P20/notes.pdf>
4. <http://www.math.iitb.ac.in/~baskar/book.pdf>
5. <https://perhuaman.files.wordpress.com/2014/07/metodos-numericos.pdf>

IV CO-CURRICULAR ACTIVITIES: (15h)

A)Mandatory:

For Teacher: Teacher shall train students in the following skills for 15 hours, by taking relevant outside data (Field/Web).

1. Applications of Newton's forward and back ward difference formulae.
2. Applications of Gauss forward and Gauss back ward, Stirling's and Bessel's formulae.
3. Applications of Newton's divided differences formula and Lagrange's interpolation formula.
4. Various methods to find the approximation of a definite integral.

5. Different methods to find solutions of Ordinary Differential Equations.

For Student: Project work

Each student individually shall undertake Project work and submit a report not exceeding 10 pages in the given format on the work-done in the areas like the following, by choosing any one of the following aspects.

1. Collecting the data from the identified sources like Census department or Electricity department, by applying the Newton's, Gauss and Lagrange's interpolation formula, making observations and drawing conclusions.

2. Selection of some region to find the area by applying Trapezoidal rule, Simpson's $1/3$ - rule, Simpson's $3/8$ - rule, and Weddle's rules. Comparing the solutions with analytical solution and concluding which one is the best method.

3. Finding solution of the ODE by Taylor's Series, Picard's method of successive approximations, Euler's method, Modified Euler's method, Runge-Kutta methods. Comparing the solutions with analytical solution, selecting the best method.

Max. Marks for Project work Report: 5.

Suggested Format for Project work Report:

Title page, Student Details, Index page, Stepwise work-done, Findings, Conclusions and Acknowledgements.

Comprehensive Continuous Assessment Test (CCIA):

(2 tests will be conducted each carries 30 Marks, consider Average Mark: 20)

B) Suggested Co-Curricular Activities:

1. Assignments, Seminar, Quiz, Group discussions/Debates.
2. Visits to research organizations, Universities, ISI etc.
3. Invited lectures and presentations on related topics by experts in the specified area.

Model Paper

Course Code: **MATSET01**

Time: 3Hrs

Offered to: B.Sc (MPC, MPCs, MECS, MSCA, MSCS, MCCS)

Semester – V/VI

Title of the Course: NUMERICAL METHODS

MAX MARKS: 70

SECTION – A

Answer any FIVE Questions

5x4=20M

1. (a) Express $f(x) = x^4 - 4x^3 + 7x^2 + 3x - 6$ in terms of the factorial notation. (CO1, L1)

(OR)

- (b) Evaluate $\Delta^n \sin(ax+b)$. (CO1, L1)

2. (a) State and prove Lagrange's interpolation formula. (CO2, L1)

(OR)

- (b) State and prove Stirling's formula. (CO2, L1)

3. (a) From the following table find the value of x for which y is minimum and find the value of y

(CO3, L2)

x	0.60	0.65	0.70	0.75
y	0.6221	0.6155	0.6138	0.6170

(OR)

- (b) Derive the formula for $\frac{dy}{dx}$ using Newton's backward interpolation formula. (CO3, L2)

4. (a) Derive Trapezoidal rule. (CO4, L2)

(OR)

- (b) Evaluate $\int_0^1 \sqrt{1+x^2} dx$ by taking $h=0.1$ using Simpson's $\frac{1}{3}$ rule. (CO4, L2)

5. (a) Solve the equation $\frac{dy}{dx} = 1 - y$ with the initial condition $y = 0$, when $x = 0$

Using Euler's algorithm and tabulate the solutions at $x = 0.1, 0.2$. (CO5, L3)

(OR)

- (b) If $\frac{dy}{dx} = x - y$; $y=1$, $x=0$ then obtain y when $x=0.5$, correct to 5 decimal places.

(CO5, L3)

SECTION – B

Answer all Questions

5x10=50M

6. (a) By using Newton's backward interpolation formula, find the value of $\tan 17^\circ$ from the following data. (CO1, L1)

θ	0°	4°	8°	12°	16°	20°	24°
$\tan\theta$							

	0	0.0699	0.1405	0.2126	0.2867	0.3640	0.4452
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(OR)

(b) State and prove the fundamental theorem of difference calculus. (CO1, L1)

7. (a) State and prove Gauss Forward interpolation formula. (CO2, L1)

(OR)

(b) Use Stirling's formula to find y_{28} , given $y_{20} = 49225$, $y_{25} = 48316$, $y_{30} = 47236$, $y_{35} = 45926$, $y_{40} = 44306$. (CO2, L1)

8. (a) Find dy/dx at $x=1$ (CO3, L2)

x	1	2	3	4	5	6
y	1	8	27	64	125	216

also find d^2y/dx^2 at $x=1$

(OR)

(b) From the Following table find x correct to two decimal places, for which y is maximum and find this value of y. (CO3, L2)

x	1.2	1.3	1.4	1.5	1.6
y	0.9320	0.9636	0.9855	0.9975	0.9996

9. (a) Find the value of $\int_0^1 \frac{dx}{1+x}$ taking 5 sub intervals by Trapezoidal rule, Correct to five Significant figures. (CO4, L2)

(OR)

(b) Find the value of $\int_4^{5.2} \log x \, dx$ by Weddle's rule. (CO4, L2)

10. (a). Given the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2+1}$ with the initial condition $y=0$, when $x=0$. Use Picard's method to obtain y for $x=0.25$, 0.5 and 1.0 correct to three decimal places. (CO5, L3)

(OR)

(b) Given $dy/dx=1+xy$ with the initial condition that $y=1$, when $x=0$. Compute y (0.1) correct to four places of decimal by using Taylor's series method. (CO5, L3).

Course Code: **MATSET02**

Offered to: B.Sc(MPC, MPCS, MECS, MSCA, MSCS, MCCS)

Domain Subject: **MATHEMATICS**

Semester – **V/VI**

Max. Marks: **100** (IA: 30+ SEE: 70)

Theory Hrs./Week: **6**

Course 7A: MATHEMATICAL SPECIAL FUNCTIONS

Type of the Course: **Skill Enhancement Course** (Elective)

Credits: **5**

I. Course Outcomes: Students at the successful completion of the course will be able to:

CO1: Acquire the information about Beta and Gamma functions, and evaluate it in various Problems.

CO2: Derive Rodrigue's formula, generating function, recurrence relations and orthogonal Property of Laguerre polynomials and use them in various applications.

CO3: Solve Hermite equation and write the Hermite Polynomial of order 'n' also find the generating function and orthogonal properties of Hermite polynomials.

CO4: Solve Legendre equation and write the Legendre equation of first kind, also find the generating function and orthogonal properties of Legendre Polynomials.

CO5: Solve Bessel's equation and write the Bessel's equation of first kind also find the generating function of Bessel's function.

II. Syllabus:

(Total Theory Hours: 75)

UNIT – I: BETA AND GAMMA FUNCTIONS, CHEBYSHEV POLYNOMIALS

(15hrs)

4.1 - Euler's Integrals-Beta and Gamma Functions, Elementary properties of Gamma Functions,

4.2 - Transformation of Gamma Functions, Another form of Beta Function,

4.3 - Relation between Beta and Gamma Functions.

4.4 - Chebyshev polynomials, orthogonal properties of Chebyshev polynomials

4.5 - Recurrence relations, generating functions for Chebyshev polynomials

UNIT –II: LAGUERRE POLYNOMIALS

(15 hrs)

- 2.1 - Laguerre's differential equation
- 2.2 - Laguerre polynomials
- 2.3 - Generating function
- 2.4 - Other forms for Laguerre polynomials
- 2.5 - Rodrigue's formula
- 2.6 - To find first few Laguerre polynomials
- 2.7 - Orthogonal properties for Laguerre polynomials
- 2.8 - Recurrence formula for Laguerre polynomials.

UNIT - III: HERMITE POLYNOMIALS

(15 hrs)

- 3.1. Hermite Differential Equations, Solution of Hermite Equation, Hermite polynomials,
- 3.2. Generating function for Hermite polynomials.
- 3.3. Other forms for Hermite Polynomials, Rodrigues formula for Hermite Polynomials, to find first few hermite Polynomials.
- 3.4. Orthogonal properties of Hermite Polynomials, Recurrence formulae for Hermite Polynomials.

UNIT – IV: LEGENDRE'S POLYNOMIALS

(15 hrs)

- 4.1. Definition, Solution of Legendre's equation, Legendre polynomial of degree n,
- 4.2. Generating function of Legendre polynomials.
- 4.3. Definition of $P_n(x)$ and $Q_n(x)$, General solution of Legendre's Equation (derivations not required) to show that $P_n(x)$ is the coefficient of h^n , in the expansion of $(1 - 2xh + h^2)^{-1/2}$.
- 4.4. Orthogonal properties of Legendre's polynomials, Recurrence formulas for Legendre's Polynomials.

UNIT – V: BESSEL'S EQUATION

(15 hrs)

- 5.1. Definition, Solution of Bessel's equation, Bessel's function of the first kind of order n,
- 5.2. Bessel's function of the second kind of order n.
- 5.3. Integration of Bessel's equation in series form $n=0$,
- 5.4. Definition of $J_n(x)$, recurrence formulae for $J_n(x)$.
- 5.5. Generating function for $J_n(x)$, orthogonally of Bessel function

III: RECOMMENDED

1. J.N. Sharma and Dr.R.K. Gupta, Differential equations with special functions, Krishna Prakashan Mandir, Meerut.

REFERENCE

2. George F. Simmons, Differential Equations with Applications and Historical Notes, Tata McGraw-Hill Edition, 1994.

WEB LINKS:

4. <https://web.mst.edu/~lmhall/SPFNS/spfns.pdf>
5. <http://www.physics.wm.edu/~finn/home/MathPhysics.pdf>
6. https://www.math.tamu.edu/~fnarc/psfiles/special_fun.pdf
7. <https://nitkkr.ac.in/docs/18-%20Series%20Solution%20and%20Special%20Functions.pdf>

IV CO-CURRICULAR ACTIVITIES: (15h)

A)Mandatory:

For Teacher: Teacher shall train students in the following skills for 15 hours, by taking relevant outside data (Field/Web).

1. Beta and Gamma functions, Laguerre polynomials.
2. Power series, power series solutions of ordinary differential equations,
3. Procedures of finding series solutions of Hermite equation, Legendre equation and Bessel's equation.
4. Procedures of finding generating functions for Hermite polynomials, Legendre Polynomials and Bessel's function.

For Student: Fieldwork/Project work:

Each student individually shall undertake Fieldwork / Project work, make observations and conclusions and submit a report not exceeding 10 pages in the given format on the work-done in the areas like the following, by choosing any one of the aspects.

1. Going through the web sources like Open Educational Resources on the properties of Beta and Gamma functions, Chebyshev polynomials, power series solutions of ordinary differential equations.
2. Going through the web sources like Open Educational Resources on the properties of series solutions of Hermite equation, Legendre equation and Bessel equation.

Max.MarksforProjectworkReport:5.

Suggested Format for Project work Report:

Title page, Student Details, Index page, Stepwise work-done, Findings, Conclusions and Acknowledgements.

Comprehensive Continuous Assessment Test (CCIA):

(2 tests will be conducted each carries 30 Marks, consider Average Mark: 20)

B) Suggested Co-Curricular Activities:

4. Assignments, Seminar, Quiz, Group discussions/Debates.
5. Visits to research organizations, Universities, ISI etc.
6. Invited lectures and presentations on related topics by experts in the specified area.

Course Code: MATSET02

Time: 3Hrs

Offered to: B.Sc(MPC, MPCS, MECS, MSCA, MSCS)

Title of the Course: MATHEMATICAL SPECIAL FUNCTIONS

MAX MARKS: 70

SECTION – A

Answer any FIVE Questions

5x4=20M

1. (a). Evaluate $\int_0^{\infty} x^2 e^{-x^2} dx$
 (OR) (CO1, L1)
 (b). P.T $\Gamma\left(\frac{3}{2} - x\right) \Gamma\left(\frac{3}{2} + x\right) = \left(\frac{1}{4} - x^2\right) \pi \sec \pi x$ where $-1 < 2x < 1$ (CO1, L1)
2. (a). Prove that $L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})$. (CO2, L2)
 (OR)
 (b). Prove that $L_n(0) = 1$.
3. (a). Prove that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$ (CO3, L3)
 (OR)
 (b). Prove that $H_n'(x) = 2nH_{n-1}(x)$ (CO3, L3)
4. (a). Prove that $P_n(-x) = (-1)^n P_n(x)$ (OR)
 (b). Prove that $(1 - x^2)P_n'(-x) = (n + 1)(xP_n(x) - P_{n+1}(x))$ (CO4, L3)
5. (a) Prove that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$ (CO5, L3)
 (OR)
 (b) State and prove Generating function for $J_n(x)$.

SECTION – B

Answer all Questions

5x10=50M

6. (a) Derive the relation between Beta and Gamma functions.
 (OR) (CO1, L1)
 (b). When n is a positive integer prove that $2^n \Gamma\left(n + \frac{1}{2}\right) = 1.3.5 \dots (2n-1) \sqrt{\pi}$.
 (CO1, L1)

7. (a). State and Prove Orthogonal properties of Laguerre polynomials. (CO2, L2)

(OR)

(b). Prove that $(n+1) L_{n+1}(x) = (2n+1-x) L_n(x) - n L_{n-1}(x)$. (CO2, L2)

8. (a). State and Prove Generating function for Hermite Polynomials (CO3, L3)

(OR)

(b). Prove that $H_n(x) = 2^n \left[\exp\left(-\frac{1}{4} \frac{d^2}{dx^2}\right) x^n \right]$. (CO3, L3)

9. (a). Prove that $(2n+1) x P_n(x) = (n+1) P_{n+1}(x) + n P_{n-1}(x)$. (CO4, L3)

(OR)

(b). State and prove Rodrigues formula for Legendre's Equation. (CO4, L3)

10. (a) Prove that $2J'_n(x) = J_{n-1}(x) - J_{n+1}(x)$. (CO5, L3)

(OR)

(b). Prove that $\sqrt{\left(\frac{\pi x}{2}\right)} J_{\frac{3}{2}}(x) = \frac{1}{x} \sin x - \cos x$. (CO5, L3).

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Course Code: **MATSET03**

Offered to: B.Sc,(MSDS)

Domain Subject: **MATHEMATICS**

Semester – **V/VI**

Max. Marks: **100** (CCIA: 30 + SEE : 70)

Theory Hrs./Week: **6**

Course 6B: MULTIPLE INTEGRALS AND APPLICATIONS OF VECTOR CALCULUS

Type of the Course: (**Skill Enhancement Course** (Elective)),

Credits: 05

I. Course Outcomes: Students at the successful completion of the course will be able to:

- CO1: Students learn about Multiple Integrals, Change of Order of Integration in Double Integral, Area and Volume by Double Integration. Triple Integrals.
- CO2: To set up and evaluate multiple integrals for regions in the plane. To find Area of the region bounded by curves and to find volume, surface area, Mass, C.G and M.I of solid geometric figures.
- CO3: Recognize vector fields and vector calculus, and define Gradient, Divergence and Curl operators.
- CO4: Compute the derivatives and line integrals, surface integrals and volume integrals of vector functions and learn their applications.
- CO5: Students learn green's theorem, Gauss Divergence theorem, Stoke's theorem and applications to evaluating line integrals and finding areas.

II. Syllabus:

(Total Theory Hours: 75)

UNIT-I: MULTIPLE INTEGRALS – I

(15 Periods)

- 1.1 Introduction, Double integrals, Evaluation of double integrals, Properties of double integrals.
1.2 Region of integration, double integration in Polar Co-ordinates,
1.3 Change of variables in double integrals, change of order of integration.

UNIT-II: MULTIPLE INTEGRALS – II

(15 Periods)

- 2.1 Triple integral, region of integration, change of variables.
2.2 Plane areas by double integrals, surface area by double integral.
2.3 Volume as a double integral, volume as a triple integral.

UNIT-III: VECTOR DIFFERENTIATION

(15 Periods)

- 3.1 Vector differentiation, ordinary derivatives of vectors.
3.2 Differentiability, Gradient, Divergence, Curl operators,
3.3 Formulae involving these operators.

UNIT-IV: VECTOR INTEGRATION

(15 Periods)

- 4.1 Line Integrals with examples.
- 4.2 Surface Integral with examples.
- 4.3 Volume integral with examples.

UNIT-V: VECTOR INTEGRATION APPLICATIONS

(15 Periods)

- 5.1 Gauss theorem and applications of Gauss theorem.
- 5.2 Green's theorem in plane and applications of Green's theorem.
- 5.3 Stokes's theorem and applications of Stokes theorem.

III References/ Text Book/ e-books/websites

1. Dr. M. Anitha, Linear Algebra and Vector Calculus for Engineer, Spectrum University Press, SR Nagar, Hyderabad-500038, INDIA.
2. Dr. M. Babu Prasad, Dr. K. Krishna Rao, D. Srinivasulu, Y. Adi Narayana, Engineering Mathematics-II, Spectrum University Press, SR Nagar, Hyderabad-500038, INDIA.
3. V. Venkateswararao, N. Krishnamurthy, B. V. S. S. Sarma and S. Anjaneya Sastry, A text Book of B.Sc., Mathematics Volume., III, S. Chand & Company, Pvt. Ltd., Ram Nagar, New Delhi-110055.
4. R. Gupta, Vector Calculus, Laxmi Publications.
5. P. C. Matthews, Vector Calculus, Springer Verlag publications.
6. Web resources suggested by the teacher and college librarian including reading material.

Reference Materials on the Web/web-links:

https://mate.unipv.it/moiola/ReaDG/VC2016/VectorCalculus_LectureNotes_2016.pdf

IV Co-Curricular Activities:

A) Mandatory:

For Teacher: Teacher shall train students in the following skills for 15 hours, by taking Relevant outside data (Field/Web).

1. The methods of evaluating double integrals and triple integrals in the class room and train to evaluate these integrals of different functions over different regions.
2. Applications of line integral, surface integral and volume integral.
3. Applications of Gauss divergence theorem, Green's theorem and Stokes's theorem.

For Student: Project work Each student individually shall undertake Project work and submit a report not exceeding 10 pages in the given format on the work-done in the areas like the following, by choosing any one of the following aspects.

1. Going through the web sources like Open Educational Resources to find the values of double and triple integrals of specific functions in a given region and make conclusions. (or)
2. Going through the web sources like Open Educational Resources to evaluate line integral, surface integral and volume integral and apply Gauss divergence theorem, Green's theorem and Stokes theorem and make conclusions.

Max.Marks for Projectwork Report:05.

Suggested Format for Project work Report:

Title page, Student Details, Index page, Stepwise work-done, Findings, Conclusions and Acknowledgements.

Comprehensive Continuous Assessment Test (CCIA):

(2 tests will be conducted each carries 30 Marks, consider Average Mark: 20)

B) Suggested Co-Curricular Activities:

7. Assignments, Seminar, Quiz, Group discussions/Debates.
8. Visits to research organizations, Universities, ISI etc.
9. Invited lectures and presentations on related topics by experts in the specified area.

Course Code: MATSET03

Time: 3Hrs

Offered to: , B.Sc(MSDS)

Title of the Course: MULTIPLE INTEGRALS AND APPLICATIONS OF VECTOR CALCULUS

MAX MARKS: 70

SECTION – A

Answer all the questions.

5 x 4 = 20 M

1. a) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dx dy$ (C01, L1)

OR

b) Evaluate $\int_0^e \int_1^{\log y} \int_1^{e^x} \log z dz dx dy$ (C01, L1)

2. a) Evaluate $\iiint xyz dx dy dz$ over the positive octant of sphere $x^2 + y^2 + z^2 = a^2$ By transforming into spherical coordinates . (C02, L1)

OR

b) Find the smaller of areas bounded by $y = 2 - x$ and $x^2 + y^2 = 4$ using double integral (C02 , L1)

3.a) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point P (1, 2, 3) in the direction of the line PQ where Q=(5, 0, 4). (C03, L2)

OR

b) Find the angle between the surfaces at the point (4, -3, 2)
 $\phi_1 : x^2 + y^2 + z^2 = 29, \phi_2 : x^2 + y^2 + z^2 + 4x - 6y - 8z - 47 = 0$ (C03, L2)

4 a) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = x^2 y^2 \vec{i} + y \vec{j}$ and the curve C is $y^2 = 4x$ in the xy - plane from (0,0) to (4,4). (C04, L3)

OR

b) Evaluate $\int_S \vec{F} \cdot N ds$, where $\vec{F} = z\vec{i} + x\vec{j} - 3y^2 z\vec{k}$ and S is the surface $x^2 + y^2 = 16$ included in the first octant between Z=0 and Z=5. (C04, L3)

5 a) Evaluate by stoke's theorem, $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ and C is the curve $x^2 + y^2 = 1, z = y^2$ (C05, L3)

OR

- b) By Using Green's Theorem, evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ and C is rectangle in xy -plane bounded by $x=0, y=0, x=a, y=a$. (C05, L3)

SECTION – B

Answer ALL the questions.

5 x 10 = 50 M

- 6 a) Change the order of integration and evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy dx$ (C01, L1)
(OR)

- b) Sketch the region of integration of $\int_a^{ae^{\pi/4}} \int_{2\log(r/a)}^{\pi/2} f(r, \theta) r dr d\theta$ and change the order of integration. (C01, L1)

- 7 a) Evaluate the triple integral $\iiint xy^2 z dx dy dz$ taken through the positive octant of the Sphere $x^2 + y^2 + z^2 = a^2$ (C02, L2)
(OR)

- b) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dx dy dz$ (C02, L2)

- 8 a) Prove that $\text{grad}(\vec{A} \cdot \vec{B}) = (\vec{B} \cdot \nabla)\vec{A} + (\vec{A} \cdot \nabla)\vec{B} + (\vec{B} \times \text{curl}\vec{A}) + (\vec{A} \times \text{curl}\vec{B})$ (C03, L2)
(OR)

- b) If \vec{a} is a constant vector, then prove that $\text{curl}\left(\frac{\vec{a} \times \vec{r}}{r^3}\right) = \frac{-\vec{a}}{r^3} + \frac{3\vec{r}}{r^5}(\vec{a} \cdot \vec{r})$ (C03, L2)

- 9 a) If $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ then evaluate $\iiint_V \phi dv$ where V is the volume enclosed by the closed region bounded by the planes $4x + 2y + z = 8, x = 0, y = 0, z = 0$ (C04, L3)
(OR)

- b) If $\phi = 45x^2y$ then evaluate $\int_S (\vec{F} \cdot \vec{N}) ds$ where S is the surface of the cube bounded by $x=0, x=a, y=0, y=a, z=0, z=a$. (C04, L3)

- 10 a) State and Prove Gauss Divergence Theorem. (C05, L3)

(OR)

- b) Verify Green's Theorem in the plane for $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the region bounded by $y = \sqrt{x}$ and $y = x^2$ (C05, L3)

Course Code: **MATSET04**

Offered to: B.Sc(MSDS)

Domain Subject: **MATHEMATICS**

Semester – **V/VI**

Max. Marks: **100** (CCIA: 30+ SEE: 70)

Theory Hrs./Week: **6**

Course 7B: INTEGRAL TRANSFORMS WITH APPLICATIONS

Type of the Course: (**Skill Enhancement Course** (Elective)),

Credits: 05

I. Course Outcomes: Students at the successful completion of the course will be able to:

CO1: Evaluate Laplace transforms of certain functions, find Laplace transforms of Derivatives and integrals.

CO2: Determine properties of Laplace transform which may be solved by application of Special functions namely Dirac delta function, error function, Bessel function and Periodic function.

CO3: Understand properties of inverse Laplace transforms, find inverse Laplace Transforms of derivatives and of integrals.

CO4: Solve ordinary differential equations with constant/ variable coefficients by using Laplace transforms method.

CO5: Comprehend the properties of Fourier transforms and solve problems related to finite Fourier transforms.

II. Syllabus:

(Total Theory Hours:75)

UNIT-I: LAPLACE TRANSFORMS – I

(15 Periods)

- 1.4 Definition of Laplace transform, linearity property-piece wise continuous function.
- 1.5 Existence of Laplace transform, functions of exponential order and of class A.
- 1.6 First shifting theorem, second shifting theorem and change of scale property.

UNIT-II: LAPLACE TRANSFORMS – II

(15 Periods)

- 2.2 Laplace Transform of the derivatives, initial value theorem and final value theorem. Laplace transforms of integrals.
- 2.3 Laplace transform of $t^n \cdot f(t)$, division by t , evolution of integrals by Laplace transforms.
- 2.3 Laplace transform of some special functions-namely Dirac delta function, error function, Bessel function and Laplace transform of periodic function.

UNIT-III: INVERSE LAPLACE TRANSFORMS

(15 Periods)

- 3.1 Definition of Inverse Laplace transforms, linear property, first shifting theorem, second shifting theorem, change of scale property, use of partial

fractions.

3.2 Inverse Laplace transforms of derivatives, inverse, Laplace transforms of integrals, multiplication by powers of 'p', division by 'p'.

3.3 Convolution, convolution theorem proof and applications.

UNIT-IV: FOURIER SERIES

(15 Periods)

4.1 Introduction, Euler's formulae for Fourier series expansion of a function $f(x)$, Dirichlet's conditions for Fourier series, convergence of Fourier series.

4.2 Functions having arbitrary periods. Change of interval, half range series.

4.3 Parseval's theorem, illustrative examples based on Parseval's theorem, some Particular series.

UNIT-V: FOURIER TRANSFORMS

(15 Periods)

5.1 Integral transforms, Fourier integral theorem (without proof), Fourier sine and Cosine integrals.

5.2 Properties of Fourier transforms, change of scale property, shifting property, Modulation theorem.

5.3 Convolution, Convolution theorem for Fourier transforms, Parseval's Identify, finite Fourier transforms.

III References/ Text Book/ e-books/websites

1. Dr.S.Sreenadh, S.Ranganatham, Dr.M.V.S.S.N.Prasad, Dr.V.Ramesh Babu, Fourier series and Integral Transforms, S.Chand & Company, Pvt. Ltd.,RamNagar,NewDelhi-110055.
2. A.R.Vasistha,Dr.R.K.Gupta,LaplaceTransforms, KrishnaPrakashanMediaPvt.Ltd.Meerut.
3. M.D.Raisinghania, H.C. Saxena , H.K. Dass, Integral Transforms, S. Chand & CompanyPvt.Ltd., Ram Nagar, New Delhi-110055.
4. Dr.J.K.Goyal,K.P.Gupta, Laplace andFourierTransforms,PragathiPrakashan, Meerut.
5. Shanthi Narayana , P.K. Mittal, A Course of Mathematical Analysis, S. Chand & Company Pvt. Ltd.Ram Nagar, New Delhi-110055.
6. Web resources suggested by the teacher and college librarian including reading material.

Reference Materials on the Web/web-links:

1. <http://aurora.phys.utk.edu/~forrest/papers/fourier/index.html> An introduction to the Fourier Transform, Fast Fourier Transform, and Discrete Fourier Transform.
2. <http://risc1.numis.nwu.edu/fft/> Public Domain code related to Fast Fourier Transforms.

IV)Co-Curricular Activities:

A) Mandatory:

For Teacher: Teacher shall train students in the following skills for 15 hours, by taking Relevant outside data (Web).

1. Demonstrate on sufficient conditions for the existence of the Laplace transform of a function.
2. Evaluation of Laplace transforms and methods of finding Laplace transforms.
3. Evaluations of Inverse Laplace transforms and methods of finding
Inverse Laplace transforms.
4. Fourier transforms and solutions of integral equations.

For Student: Project work: Each student individually shall undertake Project work and submit a report not exceeding 10 pages in the given format on the work-done in the areas like the following, by choosing any one of the aspects.

1. Going through the web sources like Open Educational Resources on Applications of Laplace transforms and Inverse Laplace transforms to find solutions of ordinary differential equations with constant/variable coefficients and make conclusions. (or)
2. Going through the web sources like Open Educational Resources on Applications of convolution theorem to solve integral equations and make conclusions. (or)
3. Going through the web source like Open Educational Resources on Applications of Fourier transforms to solve integral equations and make conclusions.

Max. Marks for Project work Report: 10.

Suggested Format for Fieldwork/Project work Report: Title page, Student Details, Index page, Stepwise work-done, Findings, Conclusions and Acknowledgements.

Comprehensive Continuous Assessment Test (CCIA):

(2 tests will be conducted each carries 30 Marks, consider Average Mark: 15)

B) Suggested Co-Curricular Activities:

1. Assignments/collection of data, Seminar, Quiz, Group discussions/Debates
2. Visits to research organizations, Universities, ISI etc.
3. Invited lectures and presentations on related topics by experts in the specified area.

Course Code: **MATSET04**

Time: 3Hrs

Offered to: B.Sc(MSDS)

Title of the Course: INTEGRAL TRANSFORMS WITH APPLICATIONS

MAX MARKS: 70

SECTION – A

Answer any FIVE of the following.

5 x 4 = 20 M

1. (a) Show that $L(\sinh at \sin at) = \frac{2a^2 p}{p^4 + 4a^4}$

(CO1,L1)

(OR)

(b) If $L[F(t)] = \frac{p^2 - p + 1}{(2p + 1)^2(p - 1)}$ then show that $L\{F(2t)\} = \frac{p^2 - 2p + 4}{4(p + 1)^2(p - 2)}$ by applying change of scale property

(CO1,L1)

2(a) Find the laplace transform of $L\left(\frac{\sin 3t \cos t}{t}\right)$

(CO2, L2)

(OR)

2.(b) Find the laplace transform of $t \sin at$ using the theorem on transforms of derivatives

(CO2,L2)

3. (a) Find the inverse laplace of $\left(\frac{6}{2p-3} - \frac{3+4p}{9p^2-16} + \frac{8-6p}{16p^2+9}\right)$

(CO3,L2)

(OR)

3.(b) Find the inverse laplace of $\frac{p-3}{p^2+4p+13}$

(CO3,L2)

4.(a) Find a_0, a_n for a Fourier Series to represent $f(x) = x^2$ in the interval $(0, 2\pi)$

(CO4, L3)

(OR)

4.(b) Find the half range sine series of $f(x) = 1$ on $[0,1]$

(CO4, L3)

5.(a) State and Prove Linear Property of Fourier Transform.

(CO5, L3)

(OR)

5.(b) Find the Fourier Sine transform of $2e^{-5x} + 5e^{-2x}$

(CO5, L4)

SECTION – B

Answer ALL the questions.

5 x 10 = 50 M

6.(a) State and Prove Second Shifting Theorem. (CO1, L2)

(OR)

6.(b) using expansion $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$ Show that $L(\sin \sqrt{t}) = \frac{\sqrt{\pi}}{2p^{3/2}} e^{-1/4p}$ (CO1, L2)

7.(a) State and Prove Initial Value theorem. (CO2, L3)

(OR)

7.(b) Using Laplace transform, evaluate $\int_0^{\infty} \frac{\cos at - \cos bt}{t} dt$ (CO2, L3)

8.(a) Find the inverse Laplace Transforms of $\left[\frac{4p+5}{(p-1)^2(p+2)} \right]$ (CO3, L4)

(OR)

8.b) State and Prove Convolution Theorem. (CO3, L3)

9.a) Find the Fourier series to represent $f(x) = x^2 - 2$, when $-2 \leq x \leq 2$ (CO4, L3)

(OR)

9.b) Find the Fourier series of $f(x) = \frac{\pi - x}{2}$ in $0 < x < 2$. (CO4, L3)

10.a) Find the Fourier transform of $f(x) = \begin{cases} 1, & \text{if } |x| < a \\ 0, & \text{if } |x| > a \end{cases}$ hence evaluate $\int_0^{\infty} \frac{\sin p}{p} dp$ and $\int_{-\infty}^{\infty} \frac{\sin ap \cos px}{p} dp$ (CO5, L4)

(OR)

10.(b) Find the Fourier sine and cosine transforms of $f(x) = \frac{e^{-ax}}{x}$ and deduce that

$$\int \frac{e^{-ax} - e^{-bx}}{x} \sin sx \, dx = \tan^{-1} \left(\frac{s}{a} \right) - \tan^{-1} (b) \quad (\text{CO5, L4})$$

SRI DURGA MALLESWARA SIDDHARTHA MAHILA KALASALA, VIJAYAWADA - 10.**(An autonomous college in the jurisdiction of Krishna University, Machilipatnam, A.P. India.)**

Course Code				23MASDL201			
Title of the Course				QUANTITATIVE APTITUDE			
Offered to: (Programme/s)							
L	2	T	0	P	0	C	2
Year of Introduction:		2024-25		Semester:			1
Course Category:				Course Relates to:			
Year of Introduction:				Percentage:		NA	
Type of the Course:				SKILL DEVELOPMENT			
Crosscutting Issues of the Course :							
Pre-requisites, if any							

Course Description:

This course aims to improve learners' mathematical and analytical abilities, particularly in the context of competitive exams, aptitude tests, and data-driven decision making. The foundational topics cover essential mathematical concepts, problem-solving techniques, and quantitative reasoning skills. The advanced topics delve into higher-level mathematics, statistical analysis, and data interpretation. Participants develop critical thinking, numerical reasoning, and logical problem-solving skills required for various professions, such as finance, consulting, and data analysis.

Course Aims and Objectives:

S. No	COURSE OBJECTIVES
1	Introduce learners to the fundamental concepts of aptitude required for recruitment processes.
2	Develop learners' problem-solving skills and critical thinking abilities in the context of recruitment aptitude tests.
3	Enhance learners' quantitative reasoning and numerical ability for solving recruitment-based problems.
4	Provide learners with strategies and techniques to improve their performance in recruitment aptitude tests.

Course Outcomes

At the end of the course, the student will / will be...

NO	COURSE OUTCOME	BTL	PO	PSO
CO1	Demonstrate a clear understanding of fundamental concepts.	K1	6	1
CO2	Apply problem-solving techniques to solve recruitment-based problems.	K2	6	2

CO3	Use appropriate strategies and shortcuts to improve speed and accuracy in solving aptitude problems during recruitment processes.	K3	6	1
CO4	Evaluate and interpret aptitude test results to identify areas of improvement and develop a personalized study plan for further enhancement.	K4	6	2
CO5	Use their logical thinking to solve Quantitative aptitude problems from company specific and other competitive test.	K5	6	1

For BTL: K1: Remember; K2: Understand; K3: Apply; K4: Analyze; K5: Evaluate; K6: Create

CO-PO-PSO MATRIX									
CO NO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PSO1	PSO2
CO1						1		1	
CO2						1			2
CO3						2		1	
CO4	✓					2			2
CO5						3		1	

Use the codes 3, 2, 1 for High, Moderate and Low correlation Between CO-PO-PSO respectively

Course Structure:

UNIT – 1: (10Periods)

Arithmetic ability: Algebraic operations BODMAS, Square roots and Cube roots, Fractions, Divisibility rules, Unit digit, Total number of factors, LCM & GCD (HCF).

UNIT – 2: (10Periods)

Quantitative aptitude: Averages, Ratio and proportion, Problems on ages, Time, distance & speed, Problems on Trains.

Business computations: Percentages, Profit & loss, Partnership, simple and compound interest, Time & work, Allegations or Mixture.

UNIT – 3: (10Periods)

Data Interpretation: Tabulation, Bar Graphs, Pie Charts, Line graphs.

Text Books:

1. Quantitative Aptitude for Competitive Examination by R.S. Agrawal, S.Chand Publications.

Reference Books:

1. Analytical skills by Showick Thorpe, published by S Chand And Company Limited, Ramnagar, NewDelhi-110055
2. Quantitative Aptitude by R V Praveen, PHI publishers.
3. Quantitative Aptitude for Competitive Examination by Abhijit Guha, Tata McGraw Hill Publications.

Links:

1. <https://www.indiabix.com/>
2. <https://www.adda247.com/>
3. [https://www.smartkeeda.com/test/Quantitative Aptitude/R Updated/all/](https://www.smartkeeda.com/test/Quantitative_Aptitude/R_Updated/all/)

❖ 15 marks for surprise tests/online tests

❖ 35 marks for semester end examination (objective type).Each question carries half mark only.

PSO1: The skills and knowledge gained has intrinsic beauty, which may leads to proficiency in analytical reasoning. This can be utilized in modelling and solving real life problems

PSO2: Handle campus placement test involving Quantitative aptitude and reasoning.

QUANTITATIVE APTITUDE

MODEL PAPER

TIME:2HRS

MAX.MARKS:35 MARKS

COURSE CODE: 23MATSDT01

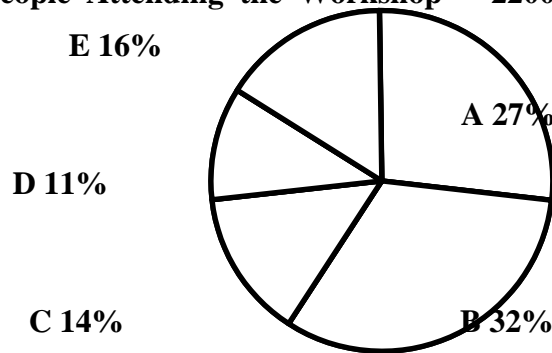
ROLL.NO:

Choose the correct answer from the following.. (70questions* 1/2 =35M)

Direction (1-5): Study the pie chart carefully to answer the given question:

Number of people attending a workshop from five different cities

Total number of People Attending the Workshop = 2200



1. What is the number of people attending the workshop from city C?L5,CO1
a) 292 b) 296 c) 322 d) 314 e) 308
2. The number of people attending the workshop from city D is what percent of the number of people attending the workshop from city E?L5,CO1
a) 125.25 b) 86.25 c) 68.75 d) 154.45 e) 145.45
3. Total number of people attending the workshop from cities A and D together is what percent of the number of people attending the workshop from city B?L5,CO1
a) 78.33 b) 84.21 c) 121.35 d) 112.25 e) 118.75
4. What is the respective ratio between the numbers of people attending the workshop from city C to those attending the workshop from city B?L5,CO1
a) 7 : 11 b) 4 : 15 c) 7 : 16 d) 14 : 23 e) 4 : 13
5. If twenty five percent of the people attending the workshop from city E are females,how many males from city E are attending the workshop?L5,CO1
a) 248 b) 90 c) 253 d) 264 e) 88
6. Find the sum of first 16 odd numbers.L3,CO2
a) 196 b) 256 c) 384 d) 424 e) None of These.

7. Find the sum of first 16 even numbers.L4,CO3

a) 240 b) 244 c) 284 d) 272 e)None of These. 8. How many numbers between 1 and 100 are such which are exactly divisible by 11?L2,CO4

a) 8 b) 9 c) 10 d) 11 e)None of These.

9. Find the square root of 2116.L1,CO5

a) 42 b) 43 c) 44 d) 45 e)None of These.

10. Find the cube root of 175616.L2,CO4

a) 56 b) 57 c) 58 d) 59 e)None of These.

11. What is 20% of 40% of 300?L4,CO3

a) 22 b) 24 c) 26 d) 28 e)None of These.

12. What is $\frac{3}{2}$ as a percentage?L3,CO2

a) 125% b) 140% c) 148% d) 150% e)None of These.

13. If a number is first increased by 20% and then decreased by 20%. What will be the net change in the number?L2,CO4

a) 5%↑ b) 4%↓ c) 4%↑ d) No change e)None of These.

14. If A: B = 4: 5 and B: C = 2: 3, then find the value of A: B: C.L2,CO4

a) 15:6:20 b) 6:15:20 c) 8:10:15 d) 14:15:12 e)None of These.

15. Find the mean proportion of 49 and 36.L4,CO3

a) 40 b) 45 c) 50 d) 58 e)None of These.

16. A and B together have Rs.2800. If $\frac{4}{11}$ of A's amount is equal to $\frac{2}{33}$ of B's amount, how much amount does B have?L3,CO2

a) Rs.2400 b) Rs.2800 c) Rs.2600 d) Rs.2200 e)None of These.

17. Find the average of first 17 natural numbers.L4,CO3

a) 7 b) 8 c) 9 d) 10 e)None of These.

18. Find the average of first seven multiples of 8.L2,CO4

a) 24 b) 32 c) 40 d)48 e)None of These.

19. The average of 14 girls and their teacher's age is 15 years. If the teacher age is excluded, the average reduced by 1 year. What is teacher's age?L2,CO4

a) 35 years b) 32years c) 29years d) 34years e)None of These.

Directions (Q.No:20 to 24): Following are the details of three shopkeepers and numbers of items sold by them on three different days:

Shopkeepers	Monday	Tuesday	Wednesday
A	160	240	210
B	200	180	320
C	150	330	280

20. Find the ratio of items sold by A and B on Monday to items sold by B and C on Wednesday?
L5,CO1

a) 5 : 3 b) 3 : 5 c) 3 : 4 d) 4 : 7 e) None of These.

21. Find the average number of items sold by all 3 shopkeepers on Wednesday?L3,CO2

a)280 b) 290 c) 270 d) 250 e) None of These.

22. Items sold by A and B together on Tuesday is what percentage of items sold by B and C on Wednesday?
L5,CO1

a)70% b) 75% c) 60% d) 65% e) None of These.

23. Find the difference of number of items sold by B on Monday and Tuesday together and items sold by A on Tuesday and Wednesday?L3,CO2

a)80 b) 60 c) 50 d) 70 e) None of These.

24. Find the ratio of items sold by B on all 3 days together to the items sold by C on all 3 days?L4,CO3

a) 35 : 38 b) 38 : 35 c) 30 : 34 d) 30 : 38 e) None of These.

25. Find the HCF of 40 and 50. L1,CO5

a) 11 b) 12 c) 10 d) 9 e)None of These.

26. Two numbers of the LCM and HCF are 360 and 30 respectively. If one of the two numbers is 120, then find the second number.L3,CO2

a) 80 b) 90 c) 140 d) 150 e)None of These.

27. Find the LCM of 20, 40 and 80.L4,CO3

a) 20 b) 40 c) 80 d) 240 e)None of These.

28. Find the simple interest on Rs.2000 at 25% per annum for 3 years.L5,CO1

a) Rs.1400 b) Rs.1800 c) Rs.1600 d) Rs.1200 e)None of These.

29. Find the compound interest on Rs.10000 at 20% per annum for 18 months. (The interest being compounded half yearly)L3,CO2

a) Rs.3320 b) Rs.3330 c) Rs.3340 d) Rs.3310 e)None of These.

30. The difference between simple interest and compound interest at 10% per annum for 2 years is Rs.22. Find the sum.L4,CO3

a) Rs.2200 b) Rs.2300 c) Rs.2400 d) Rs.2500 e)None of These.

31. A man bought an article at Rs.1600 and sold for Rs.2100. Find the loss or profit.L1,CO5

a) Rs.400 (loss) b) Rs.800 (profit) c)Rs.500(profit) d)Rs.1200(loss) e)None of These.

32. A dishonest dealer sells his goods at cost price but he uses 600 grams instead of 1 kg. Find the profit percentage.L3,CO2

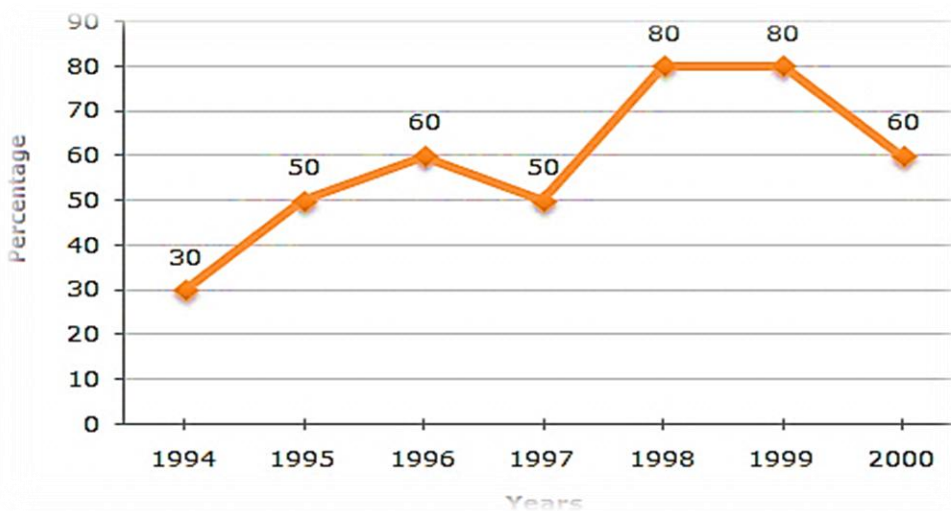
a) 20% b) 25% c) 33.33% d) 35% e)None of These.

33. The selling price of two books is Rs.2000 each. If one book sold at a gain of 20% profit and the other at a loss of 20%, then find the profit or loss percentage in the deal. L2,CO4

a) 5% profit b) 2% loss c) 4% loss d)3%profit e)None of These.

Directions(Q.No-34 to 38): The following line graph gives the percentage of the number of candidates who qualified an examination out of the total number of candidates who appeared for the examination over a period of seven years from 1994 to 2000.

Percentage of Candidates Qualified to Appeared in an Examination Over the Years



34. The difference between the percentages of candidates qualified to appeared was maximum in which of the following pairs of years?L2,CO4

a) 1994 and 1995 b)1997 and 1998 c)1998 and 1999 d)1999 and 2000 e) None of these

35. In which pair of years was the number of candidates qualified, the same?L3,CO2

a) 1995 and 1997 b) 1995 and 2000 c) 1998 and 1999 d) Data inadequate e) None of these

36. If the number of candidates qualified in 1998 was 21200, what was the number of candidates appeared in 1998?L4,CO3

a) 32000 b) 28500 c) 26500 d) 25000 e) None of these

37. If the total number of candidates appeared in 1996 and 1997 together was 47400, then the total number of candidates qualified in these two years together was?L5,CO1

a) 34700 b) 32100 c) 31500 d) Data inadequate e) None of these

38. The total number of candidates qualified in 1999 and 2000 together was 33500 and the number of candidates appeared in 1999 was 26500. What was the number of candidates in 2000? L4, CO3

- a) 24500 b) 22000 c) 20500 d) 19000 e) None of these

39. Find the least value of '*' so that the number $356*8$ is divisible by 4. L3, CO2

- a) 0 b) 1 c) 2 d) 3 e) None of These.

40. $144 + 72 \div 12 - 5 \text{ of } 2 = ?$ L4, CO3

- a) 130 b) 110 c) 100 d) 150 e) None of These.

41. Find the least value of '*' so that the number $356*3$ is divisible by 3. L2, CO4

- a) 0 b) 1 c) 2 d) 3 e) None of These.

42. $45 + 3 \times 2 - 35 \div 7 \text{ of } 5 = ?$ L1, CO5

- a) 60 b) 50 c) 43 d) 46 e) None of These.

43. Find the least value of '*' so that the number $3565*$ is divisible by 5. L3, CO2

- a) 0 b) 1 c) 2 d) 3 e) None of These.

44. $56 + 36 \div 9 \text{ of } 4 - 3 \times 2 = ?$ L1, CO5

- a) 40 b) 50 c) 60 d) 30 e) None of These.

45. If the capitals of P & Q are in the ratio of 15:13 and the times of their investments are in the ratio 26:45. Then find their Profits Ratio? L5, CO1

- a) 14:15 b) 4:5 c) 2:3 d) 5:9 e) None of These.

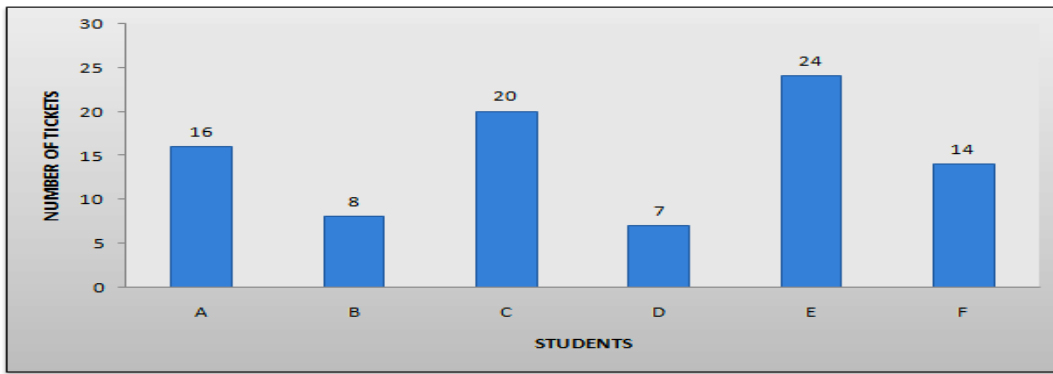
46. In a business A, B and C invested Rs.5000, Rs.6000 & Rs.7000 respectively. Find the share of A in the total profit of Rs.5400. L2, CO4

- a) Rs.1000 b) Rs.1800 c) Rs.1400 d) Rs.1500 e) None of These.

47. P and Q started a retail store with initial investments in the ratio 5 : 6 and their annual profits were in the ratio 2 : 3. If P invested the money for 8 months. For how many months did Q invest his money? L3, CO2

- a) 5 months b) 6 months c) 8 months d) 10 months e) None of These.

Directions(Q.No-48 to 52): The bar graph, given here, shows the number of tickets sold by 6 students A, B, C, D, E and F during a fair. Observe the graph and answer questions based on it.



48. Total number of tickets sold by A, B and C is..L3,CO2
 a) 45 b) 44 c) 42 d) 40 e) None of these
49. The least number of tickets were sold by..L2,CO4
 a) B b) F c) A d) D e) None of these
50. Total number of tickets sold by D, E and F is..L3,CO2
 a) 47 b) 46 c) 45 d) 44 e) None of these
51. Find the increase percentage of the tickets from B to C.L4,CO3
 a) 145% b) 140% c) 150% d) 155% e) None of these
52. Find the increase percentage of the tickets from D to C.L4,CO3
 a) 60% b) 62% c) 64% d) 66% e) None of these
53. What is the unit digit of $1437 + 2437 - 1723$?L5,CO1
 a) 1 b) 2 c) 3 d) 4 e) None of These.
54. Find the total number of factors of 28.L3,CO2
 a) 6 b) 8 c) 12 d) 10 e) None of These.
55. What is the unit digit of $1235 \times 2751 \times 4176$?L2,CO4
 a) 1 b) 2 c) 8 d) 4 e) None of These.
56. Find the total number of factors of 56.L1,CO5
 a) 6 b) 7 c) 8 d) 9 e) None of These.
57. What is the unit digit of $2346^{182632567}$?L3,CO2
 a) 5 b) 6 c) 7 d) 8 e) None of These.
58. Find the total number of factors of 169.L4,CO3
 a) 1 b) 2 c) 3 d) 4 e) None of These.
59. The ratio of the present ages of Rahul and Reena is 5:3. After 5 years, their ratio will be 20: 13. At the time of their marriage, their ratio was 15: 8. How many years ago, they got married?L3,CO2

- a) 6yrs b) 3yrs c) 5yrs d) 7yrs e)None of These.

60. Four years ago, the ratio between the ages of Ramesh and Rajesh was 5:1. Five years hence, the ratio between their ages will be 7: 2. What is the present age of Ramesh?L5,CO1

- a) 24yrs b) 28yrs c) 32yrs d) 36yrs e)None of These.

61. A car covers a distance of 540 km in 9 hours. What is the average speed of the car?L3,CO2

- a) 55kmph b) 65kmph c) 66kmph d) 80kmph e)None of These.

62. A car travelling at a speed of 63 km/hr can complete a journey in 10 hours. How long will it take to travel the same distance at 70 km/hr?L4,CO3

- a) 6 hours b) 8 hours c) 7 hours d) 9 hours e)None of these.

63. A 560 m long train crosses a pole in 70 seconds. What is the speed of the train?L5,CO1

- a) 12 mps b) 5 mps c) 6 mps d) 8 mps e)None of These.

64. How long does a train 630 m long running at the rate of 54 kmph take to cross a tunnel 180 m in length?L4,CO3

- a) 48 sec b) 54 sec c) 40 sec d) 45 sec e)None of These.

65. A can do a piece of work in 40 days while B can do it in 80 days. In how many days can A and B working together does it?L3,CO2

- a)11 $\frac{1}{3}$ days b)12 $\frac{1}{5}$ days c)26 $\frac{2}{3}$ days d)13 $\frac{1}{9}$ days e)None of these

66. A can complete the work in 15 days, B in 20 days, C in 30 days. All started work together. But after 3 days, A left. After 5 more days B also left the work. In how many days C can complete the remaining work?L5,CO1

- a)4 days b)6 days c)3 days d)5 days e) None of These.

67. A and B can together finish a work 40 days. They worked together for 20 days and then B left. After another 30 days, A finished the remaining work. In how many days A alone can finish the work?L5,CO1

- a)40 days b)60 days c)30 days d)50 days e) None of These.

68. The amount of water (in ml) that should be added to reduce 7 ml lotion, containing 70% milk, to a lotion containing 35% milk, is:L5,CO1

- a) 10 ml b) 20ml c) 16 ml d) 7 ml e)None of These.

69. In what ratio must a grocer mix two varieties of pulses costing Rs. 14 and Rs. 19 per kg respectively so as to get a mixture worth Rs. 16.50 kg?L4,CO3

- a) 7:3 b) 1:2 c) 3:4 d) 4:7 e)None of These.

70. A grocer wishes to sell a mixture of two varieties of pulses worth Rs.18 per kg. In what ratio must he mix the pulses to reach this selling price, when cost of one variety of pulses is Rs.14 per kg and the other is Rs.24 per kg?L3,CO2

- a) 2:3 b) 4:1 c) 1:4 d) 3:2 e)None of These.

Course Code							
Title of the Course				REASONING			
Offered to: (Programme/s)							
L	2	T	0	P	0	C	2
Year of Introduction:		2024-25		Semester:			1
Course Category:				Course Relates to:			
Year of Introduction:				Percentage:		NA	
Type of the Course:				SKILL DEVELOPMENT			
Crosscutting Issues of the Course :							
Pre-requisites, if any							

Course Description:

This course aims to improve learners' mathematical and analytical abilities, particularly in the context of competitive exams, aptitude tests, and data-driven decision making. The foundational topics cover essential mathematical concepts, problem-solving techniques, and quantitative reasoning skills. The advanced topics delve into higher-level mathematics, statistical analysis, and data interpretation. Participants develop critical thinking, numerical reasoning, and logical problem-solving skills required for various professions, such as finance, consulting, and data analysis.

Course Aims and Objectives:

S. No	COURSE OBJECTIVES
1	Introduce learners to the fundamental concepts of aptitude required for recruitment processes.
2	Develop learners' problem-solving skills and critical thinking abilities in the context of recruitment aptitude tests.
3	Enhance learners' quantitative reasoning and numerical ability for solving recruitment-based problems.
4	Provide learners with strategies and techniques to improve their performance in recruitment aptitude tests.

Course Outcomes

At the end of the course, the student will / will be...

NO	COURSE OUTCOME	BTL	PO	PSO
CO1	Develop the Analytical skills to understand alphabet test, alpha numeric sequence and logical sequence test.	K1	6	1

CO2	Analyze the given number or letter to find out hidden analogy and apply that analogy to find solutions .finding odd man out by observing the principle which makes the other similar.	K2	6	2
CO3	Construct mind maps of coding and decoding of various mathematical elements	K3	6	1
CO4	Apply the basic functionality of clocks and calendar to find the solution for the problem	K4	6	2
CO5	Facilitating analytical reasoning and logical reasoning skills	K5	6	1

For BTL: K1: Remember; K2: Understand; K3: Apply; K4: Analyze; K5: Evaluate; K6: Create

CO-PO-PSO MATRIX									
CO NO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PSO1	PSO2
CO1						1		1	
CO2						1			2
CO3						2		1	
CO4	✓					2			2
CO5						3		1	

Use the codes 3, 2, 1 for High, Moderate and Low correlation Between CO-PO-PSO respectively

Course Structure:

UNIT – 1:

(10Periods)

- Number Series
- Letter series
- Odd man out
- Analogy
- Alpha-numeric series.

UNIT – 2:

(10Periods)

- Coding &Decoding
- Blood relationship
- Clocks
- Calendars.

UNIT – 3:

(10Periods)

- Directions
- Seating arrangements
- Puzzles
- Ranking test.

Text Books:

1. Verbal Reasoning for Competitive Examination by R.S. Agarwal, S.Chand publications.

Reference Books:

1. Arihant's A New Approach to Reasoning by BS Sijwali, InduSijwali.
2. Quantitative Aptitude and Reasoning by R V Praveen, PHIpublishers.
3. Arihant's How to crack test of Reasoning by Jaikishan, Premkishan.

Links:

4. <https://www.indiabix.com/>
5. <https://www.adda247.com/>
6. https://www.smartkeeda.com/test/Quantitative_Aptitude/R_Updated/all/

❖ 15 marks for surprise tests/online tests

❖ 35 marks for semester end examination (objective type).Each question carries half mark only.

PSO1: The skills and knowledge gained has intrinsic beauty, which may leads to proficiency in analytical reasoning. This can be utilized in modelling and solving real life problems

PSO2: Handle campus placement test involving Quantitative aptitude and reasoning.

REASONING
MODEL PAPER

TIME: 2 HRS

MAX MARKS: 35M

COURSE CODE: 23MATSDT02

Roll no:

Choose the correct answer from the following.. (70questions* ½ =35M)

Directions (Q.No-1to5): Study the following information and answer the question:

Eight friends, A, B, C, D, J, K, L and M are seated in a straight line, facing North, but not necessarily in the same order.

* Only three people sit between C and L. L sits at one of the extreme ends of the line.

* D sit second to left of M. M is not an immediate neighbour of C. Only three people sit between M and K. A is one of the immediate neighbours of K. B is neither an immediate neighbour of L nor M.

1. Who amongst the following sit exactly between K and J? L5,CO1

- a) B, D b) A, B c) A, C d) A, L e) C, D

2. Based on the given arrangement, which of the following is true with respect to J? L4,CO2

- a) Only three persons sit between J and D b) Both A and K are immediate neighbours of J
c) M sits to immediate left of J d) J sits at one of the extreme ends of the line
e) None of the given options is true

3. How many persons are seated between B and M? L3,CO2

- a) Two b) Three c) Four d) Five e) None

4. What is the position of M with respect to C?

- a) Fourth to the right b) Second to the left c) Fourth to the left d) Third to the right
e) Second to the right

5 Who amongst the following persons are at extreme ends of the line?

- a) B, L b) A, B c) A, C d) A, L e) C, D

Directions (Q.NO- 6to10): Complete the Series. L3,CO2

6. 4, 8, 11, 22, 25, 50, 53, __.

- a)104 b)103 c)104 d)106 e)None of These.

7. 15, 17, 20, 24, 29, __.

- a)38 b)40 c)50 d)52 e)None of These.

8. 200, 100, 50, 25, __.

- a)10.5 b)12.5 c)21 d)25 e)None of These.

9. 11, 28, 327, 464, __.

- a)2348 b)5245 c)5125 d)1050 e)None of These.

10. 14, 25, 15, 23, 16, 21, 17, __.

- a)17 b)15 c)14 d)19 e)None of These.

Directions (Q.No 11to15):

In each of the following questions given below, a group of digits/letters is given followed by four combinations of symbols numbered A, B, C and D. you have to find out which of the following four combinations correctly represents the group of digits/letters based on the symbol codes and the conditions given below.

Letter	A	F	I	W	S	E	M	R	C	U	H	P	X	Y	J	G	O
Code	9	!	5	2	1	#	6	\$	%	7	&	3	4	?	0	8	@

11. What would be the code of the word ‘XUAFM’? L4,CO2

- a)9@0?3 b)&9?25 c)469!7 d)479!6 e)None of these

12. What would be the code of the word ‘YEGIM’?L3,CO2

- a)85@#& b)?#856 c)88 d)8@#&@ e)None of these

13. What would be the code of the word ‘SFWJC’?

- a)1!20% b)08%1@ c)03%1% d)301@% e)None of these

14. What would be the code of the word ‘IUEAH’? L3,CO2

- a)#9?4\$ b)?659# c)\$954? d)57#9& e)None of these

15. What would be the code of the word ‘HCYUX’? L4,CO2

- a)6%792 b)#6%92 c)6##&2 d)2%769 e)None of these

Directions (Q.No-16to20): complete the series L4,CO2

16. BR, DT, HV, NX, __.

- a)UY b)UZ c)VZ d)VY e)None of These.

17. G, L, Q, V, __.

- a)A b)B c)C d)D e)None of These.

18. C, E, H, J, M, O, __.

- a)V b)U c)S d)Y e)None of These.

19. M, N, O, L, Q, J, __.

- a)H b)S c)Z d)E e)None of These.

20. J, L, O, S, __.

- a)W b)T c)X d)J e)None of These.

Directions (Q.No-21to22):: Study the information given below carefully and accordingly answer the following questions:

Akash is going shopping at a mall nearby with his friends. They start from point X and move 15 km to the north. They then take a right turn and move 20 km, followed by a left turn. After moving for another 40 km, they turn left and walk 20 km.

21. In which direction is Akash along with his friends with reference to point X? L3,CO2

- a) North b) South c) East d) West e) North- East

22. What is the distance between point X and the final destination?

- a) 45 km b) 35 km c) 65 km d) 55 km e) 75 km

Directions (Q.No-23to25): Read the following information carefully and answer the questions given below.

Letter	Q	W	E	R	T	Y	U	I	O	P
Code	1	@	3	\$	%	5	&	8	*	9

23. What would be the code of the word 'UEIW'? L3,CO2

- a) &39\$ b) &38@ c) %581 d) 35@* e) None of These.

24. What would be the code of the word '1358'?

- a) YUIO b) WRUI c) QEYI d) TRSL e) None of These.

25. What would be the code of the word 'OPTY'?

- a) 3\$56 b) 5%6& c) 1@3\$ d) *9%5 e) None of These.

Directions (Q.No-26to30): Analyze the elements. L5,CO1

26. N/O: 41/51:: X/V: __

- a) 40/22 b) 41/22 c) 42/22 d) 43/22 e) None of These.

27. 8:64:: 10: __

- a) 100 b) 121 c) 144 d) 169 e) None of These.

28. RIN:41:: VIM: __.

- a) 64 b) 44 c) 23 d) 31 e) None of These.

29. Quarrel : War :: Unhappy: __

- a) Happy b) Sad c) Refuse d) Deny e) None of These.

30. 245:40:: 243: __

- a) 19 b) 24 c) 21 d) 17 e) None of These.

Directions (Q.No-31to35): Study the following sequence of numbers and alphabets and answer the given questions-

Q 2 M © W N 9 E B @ R 5 V T \$ C 3 Y & A S 7 % P Z F 8 K U * O 6 G £ H

31. How many such letters are there in the given series which are immediately preceded by a symbol and immediately followed by a number? L3,CO2

- a) Two b) Four c) One d) Three e) None

32. If all the symbols are dropped in the given series, which element will be eleventh from the left end?

- a) V b) 5 c) T d) 7 e) C

33. Which of the following element is fourth to the left of the ninth element from the right end of the given arrangement?

- a) % b) S c) P d) K e) &

34. If all the digits are deleted from the given arrangement, which of the following will be eighth from the left end of the arrangement?L3,CO2

- a) W b) E c) * d) @ e) R

35. Which of the following is third to the right of the fourteenth digit from the right end of the given arrangement?

- a) % b) Z c) 3 d) 7 e) K

Directions (Q.No-36 to 40): Find the odd thing in. L5,CO1

36. a)13 b)23 c)33 d)43 e)53
37. a)25 b)36 c)49 d)64 e)81
38. a)C b)F c)I d)L e)O
39. a)Desk b)Blackboard c)Classroom d)Bench e)Chalk
40. a)248 b)326 c)414 d)392 e)428

41. Find the mirror image of 3:20 am.

- a)9:40 pm b)8:40 pm c)7:40 pm d)6:40 pm e)None Of These

42. Find the mirror image of 9:30.L3,CO2

- a)9:40 b)2:30 c)7:40 d)3:30 e)None Of These

43.Howmany times the hands of a clock are right angles in 24 hours?

- a)44 b)11 c)24 d)12 e)None Of These

44.Find the angle between minute hand and hour hand, when the time is 3:22?L3,CO2

- a)81⁰ b)61⁰ c)31⁰ d)71⁰ e)None Of These

45.At what point of time after 3 O'clock, will the hands of a clock be in the right angles for the first time?L5,CO1

- a)30 2/11 min b) 31 3/11min c)322/11min d)333/11min e)None Of These

46. March7th 2023 was Tuesday. What day of the week will it be on November 15th2023?

- a)Wednesday b) Tuesday c) Sunday d) Saturday e) None of these.

47. The year next to 2046 having same calendar as that of 2046 is.L3,CO2

- a)2057 b) 2056 c) 2061 d) 2058 e) None of these.

48. Find the number of odd days in 65 days.L3,CO2

- a)1 b)6 c)2 d)5 e) None of these.

49. If today is Thursday after 71 days it will be.

- a) Sunday b) Saturday c) Monday d) Tuesday e) None of these.

50. What will be the day of the week on 26th January, 2024? L5, CO1

- a) Sunday b) Thursday c) Friday d) Tuesday e) None of these.

Directions (Q.No-51 to 55): complete the series. L5, CO1

51. Z-25, X-23, V-21, T-19, ___.

- a) R-17 b) F-14 c) N-14 d) M-14 e) None of these.

52. 5A, 8B, 11C, 14D, ___.

- a) 15F b) 17E c) 18F d) 20L e) None of these.

53. C4Z, F8X, I12V, ____, O20R.

- a) L13S b) L15S c) I19S d) L12T e) None of these.

54. X-144, ____, T-100, R-81, P-64.

- a) T-121 b) U-122 c) T-123 d) V-124 e) None of these.

55. J6R, H8Q, ____, D15O, B20N.

- a) I9L b) L9S c) I9S d) F10P e) None of these.

Directions (Q.No-56 to 60): Study the following information and answer the given Question:

K is father of Y. Y is brother of G. L is married to G. L is daughter of X. B is Wife of X. P is son of B. Z is daughter of P. C is daughter in law of X. B has only one son.

56. How is G related to K? L3, CO2

- a) Niece b) Son c) Nephew d) None e) Cannot be determined

57. How is G related to B?

- a) Son-in-law b) Son c) Uncle d) Nephew e) Brother

58. If S is married to Z, then how is C related to S? L3, CO2

- a) Mother-in-law b) Son c) Daughter d) Father-in-law e) Cannot be determined

59. How is L related to K?

- a) Niece b) Daughter-in-law c) Mother-in-law d) Sister e) None.

60. How is Z related to L? L3, CO2

- a) Niece b) Daughter c) Nephew d) None e) Cannot be determined

Directions (Q.No-61 to 65): Study the following information carefully and answer the given question.

Eight people- A, B, C, D, E, F, G and H are sitting around a circular table (facing the centre) with equal distances between each other, but not necessarily in the same order.

C sits third to the right of G. Only one person sits between G and F. A sits to the immediate left of F. B is neither an immediate neighbour of G nor C. H sits second to the left of B. D is one of the immediate neighbours of H.

61. Which of the following statements is **NOT TRUE** with respect to the given arrangement?L3,CO2

- a) B is an immediate neighbour of both E and D b) C sits second to the left of D c) C sits to the immediate right of F d) Only three people sit between A and E e) All the given options are true

62. Who amongst the following sit/s exactly between F and D. when counted from the left of D?

- a) Both C and H b) Only E c) Only C d) Both H and A e) Both A and G

63. If all the people are made to sit in alphabetical order in anti-clockwise direction starting from A, the positions of how many people (excluding A) will remain unchanged?L3,CO2

- a) Three b) One c) None d) Two e) More than three

64. Who amongst the following sits to the immediate right of E?

- a) H b) C c) B d) A e) G

65. Who amongst the following sits second to the right of H?L3,CO2

- a) D b) C c) B d) A e) G

Directions(Q.No-66to68): Study the following information carefully and answer the questions given below.

Colony U, V, W, X, Y and Z has different number of houses. Only Colony Z has more number of houses than Colony X. Colony V has more number of houses than Colony Y but less than Colony U. Colony W has more number of houses than both Colony Y and Colony U. The colony having the third highest number of houses is 39. Colony Y has 24 houses.

66. If the sum of the number of houses in colony W + Z is Sixty six more than the number of houses in Colony Y, how many of houses are there in Colony Z?

- a) 31 b) 46 c) 51 d) 55 e) 45

67. How many houses do Colony V probably has?L3,CO2

- a) 39 b) 43 c) 55 d) 31 e) 14

68. Which of the following is true regarding the number of houses in colony 'U'?L3,CO2

- a) No other colony has less houses than U. b) X has more number of houses than U.
c) U possibly has 45 houses. d) U has more number of houses than only three colonies.
e) None of the given options is true

69. Lalit along with his family decided to take a road trip to a nearby resort and spend the weekend there. He started from his home and from there drove 70 km to the south, he then took a right turn and drove 30 km. Next, he took a right turn and drove 30 km and stopped at a restaurant. What is the shortest distance between his house and the restaurant? L5,CO1

- a) 75 km b) 35 km c) 95 km d) 25 km e) None of these.

70. A man is facing west. He turns 45° in the clockwise direction and then another 180° in the same direction and then 270° in the anti-clockwise direction. Which direction is he facing now? L3,CO2

- a) North b) South –West c) South d) East e) None of these.